

Making Retirement Income Last a Lifetime

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The goal of this article is to explore the sustainability of investment portfolio withdrawals using two distinct methodologies—historical analysis and Monte Carlo simulations—to address the risk of extreme longevity. The article also examines whether annuitizing a portion of client assets makes it more likely that retirees can enjoy higher incomes over longer retirements.

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What portfolio withdrawal rate should advisors recommend during the retirement years—and how can they make the resulting income flows last as long as possible?

The reliability of professional retirement planning advice depends to a great extent on the integrity of one number: the amount of an investment portfolio that can be safely liquidated each year to provide income in retirement. That estimated number, expressed either in dollars or as a percentage of assets, fixed or (preferably) inflation-adjusted, determines the optimal size of a client's target retirement portfolio and therefore also defines the entire accumulation process during the working years. On the other side of the retirement line, one needs to know how much income can be taken from a portfolio without leaving retirees broke or destitute in their later years.

Unfortunately, professional advisors don't have certain knowledge of future investment returns, inflation rates or the length of their clients' lives. They are, therefore, required to make educated guesses about how much income a retirement portfolio can ultimately provide.

The goal of this paper is to explore the issue of portfolio liquidation using a combination of methods and tools, from simple analysis of past experience to Monte Carlo simulations of many thousands of hypothetical future scenarios. In addition, we explore several possible ways to make the liquidation process more stable and certain—in other words, to increase the likelihood that an initial portfolio liquidation rate can be sustained all the way out to the end of a retiree's life.

Some of this territory has already been explored in the professional literature. In his [1994 article](#) for this publication, financial advisor Bill Bengen started with a compelling scenario. He noted that *Money* magazine had recommended that investors spend 5.29 percent of their portfolios in the first year of retirement. The magazine also recommended that, in subsequent years, they take out the inflation-adjusted equivalent of this figure (keeping constant the purchasing power of the amount liquidated from the portfolio) until death. Given the double-digit yearly returns that stock portfolios have provided since 1926, this 5.29 percent figure might appear to be reasonable and even conservative. But when Bengen performed a simple spreadsheet analysis using the historical returns on the portfolios with asset mixes that he, himself, was recommending, and assumed that a client had retired in 1972, he found that the client would have completely run out of money after 23 years. A client retiring in 1966, using the same strategy, would have been bankrupted after 18 years.¹ Clearly, if past conditions repeated themselves, people who took *Money* magazine's advice could have found themselves in financial difficulty.

In [follow-up analyses](#), Bengen found that if he changed the asset allocation mix, he changed the sustainable 30-year withdrawal rates, which tended to peak at around 4.3 percent at stock allocations of between 50 percent and 75 percent.² When he looked at rolling 30-year periods taken quarterly rather than annually, he found that the highest sustainable withdrawal rates were almost exactly the same, but were experienced in portfolios whose stock allocations fell in a narrower range between 55 percent and 65 percent.³

A subsequent analysis by Gordon Pye offered similar but not identical results. Instead of using actual historical returns, Pye created a Monte Carlo simulation model, which evaluated possible sequences of future returns on various asset categories. The Monte Carlo simulation tool created by Pye basically takes the range and standard deviation of historical returns and inflation and simulates possible future outcomes of these variables. Over a relatively short ten-year retirement period, Pye found that a stock portfolio was able to sustain withdrawals of four percent of the initial portfolio, inflation-adjusted, in 92 percent of the hypothetical future sequences of returns. When the analysis was extended to 20-year and 35-year time horizons, the 4 percent liquidation rate succeeded more than 80 percent of the time.⁴

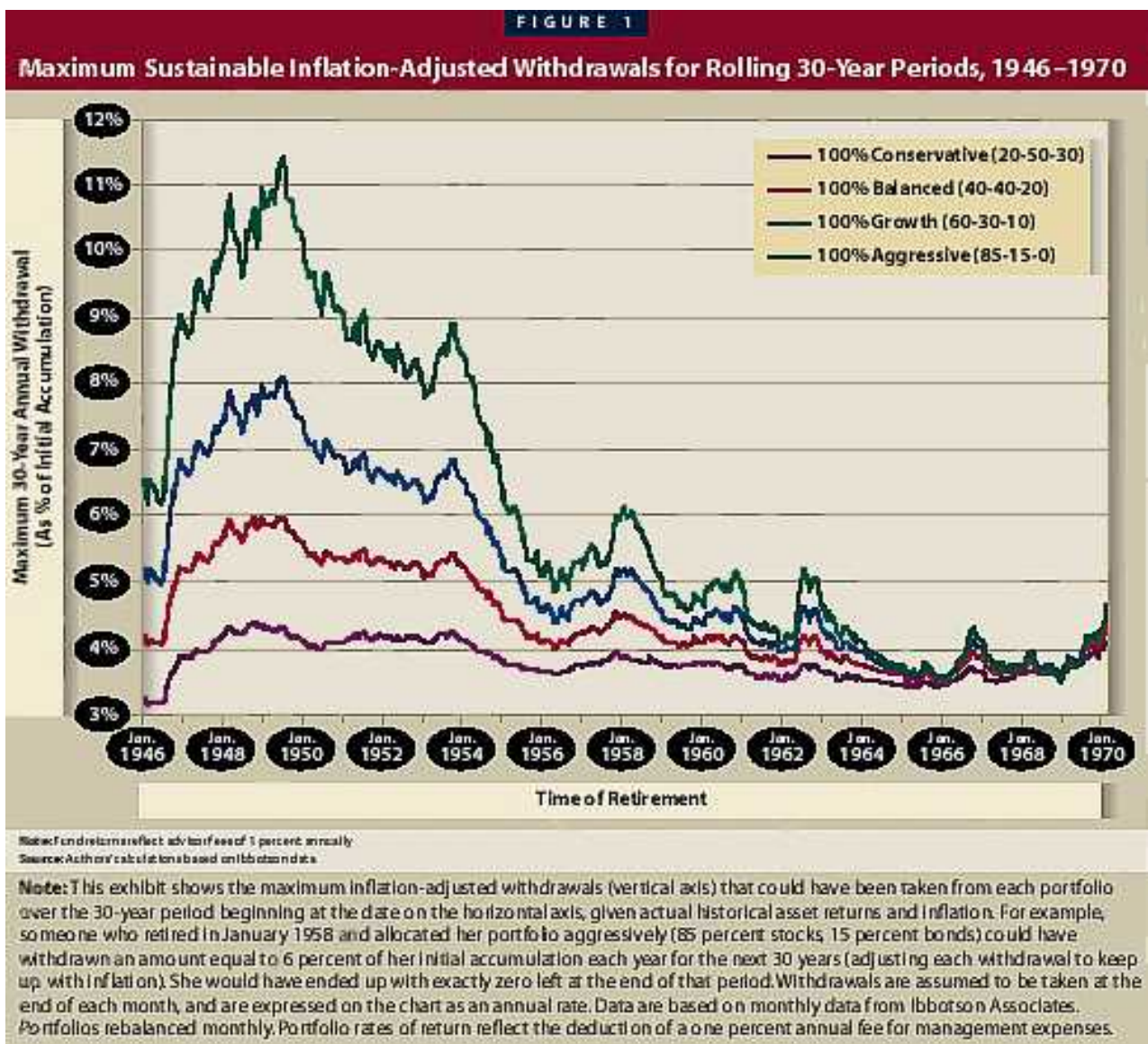
As illustrated by these analyses, it appears that investors cannot be confident of liquidating more than four percent (inflation-adjusted) of their retirement portfolios. It also appears that results derived from analysis of the historical sequence of actual returns may be overly optimistic if one admits the possibility for a broader range of possible future return sequences.

These analyses raise some questions. Is it possible to maintain withdrawal rates higher than about four percent, inflation-adjusted, in retirement? By varying the mix of assets or by introducing other types of investments into the portfolio, can we make the yearly withdrawals more sustainable? Also, given the trend toward earlier retirement dates and longer life expectancies, and planning for a couple rather than an individual, how confident are we that a 30-year time horizon is long enough? In retirement planning work, shouldn't we consider sustainable withdrawal rates over longer time horizons?

Past as Prologue

To start our own evaluation of portfolio liquidation options, we first define a range of investment portfolios for evaluation. The analysis that follows is based on four different asset portfolios of retirees, defined by their holdings of, respectively, stocks (represented by the S&P 500), bonds (represented by intermediate-term government bonds) and cash (30-day T-bills). We will hereafter refer to a "conservative" portfolio defined as one that holds 20 percent stocks, 50 percent bonds and 30 percent cash; a "balanced" portfolio with a 40-40-20 mix; a "growth" portfolio allocated 60-30-10; and an "aggressive" portfolio allocated 85-15-0. (All return data are from Ibbotson Associates.)

The reader's eye is directed to Figure 1, which shows the amount that could have been liquidated from portfolios over rolling monthly 30-year time periods. The figures shown are based on actual historical total returns and inflation; the only adjustment is that we assessed a one percent annual portfolio management fee to cover the costs of professional management and advice.⁵ In Figure 1, we allowed each portfolio to grow at the rate dictated by the historical monthly returns of each of its asset components. We then calculated how much of the original value of the portfolio, expressed as a percentage, could have been taken out each year (in constant purchasing power) so that the portfolio would be completely extinguished after 30 years. Thus, if the figure were ten percent of a \$1 million portfolio, then the income figure in the first year would have been \$100,000 a year (\$1 million times ten percent). For each subsequent year, we adjusted the dollar amount being liquidated from the portfolio to account for effects of inflation.



This calculation was made for retirees who left work in January of 1946 and died in December of 1976, and then again for retirees who retired in February of 1946 and died in January of 1977, and for rolling-monthly 30-year periods thereafter. The calculations in Table 1 end with persons who retired in January of 1970 and extinguished their portfolio with their last income check in December 1999.

Figure 1 shows how much this "liquidation rate" can fluctuate over time. A retiree holding an aggressive portfolio could have retired in the summer months of 1949 and liquidated the inflation-adjusted equivalent of more than 11 percent of the initial amount of the portfolio each year, for the next 30 years. A retiree holding a conservative portfolio, by contrast, could have barely taken out the inflation-adjusted equivalent of four percent of the original portfolio for the following 360 months.

It is natural, when looking at graphs like Figure 1, for the reader's eye to default to the top lines and calculate the maximum liquidation that was possible with each of the four portfolios. But an equally important issue is the variability of that portfolio liquidation factor. After all, retirees standing on the threshold of each of these "conceptual" futures had no way of knowing what this graph line would say about their future prosperity.

We can see that the aggressive portfolio offered considerably more variability of possible liquidation rate than any of the other investment mixes. When compared with the alternatives, however, we find that most of the variability was on the upside. Although the range of possible liquidation rates was much broader with the aggressive portfolio than the less-volatile portfolios, the aggressive portfolio almost always outperformed the alternatives as an income-producing

vehicle, and where it did underperform, the difference was relatively small. The same is generally true of the growth portfolio compared with the balanced, and the balanced compared with the conservative. Although investors may have experienced very different income results over different time periods, they were almost always better off financially with the more volatile portfolios than with the portfolios that offered more year-to-year certainty but generally lower returns. These results, of course, are a reflection of the historical return premium on equities. There is no guarantee that this pattern will repeat itself into the future.

Also striking about Figure 1 is how all of the four (very different) asset mixes converge sometime in early 1965—and how the convergence persists through the end of the graph. Here we are seeing the effect of the timing of investment returns. The retirees who left work at any point in calendar 1964 through calendar 1969 (when the graph runs out of 30-year periods to measure) would have experienced the 1973–74 bear market early in their retirement. The earlier significant negative returns hit a retirement portfolio, the harder it is to sustain higher liquidation rates, even though, as was the case for people retiring in the late 1960s, they ultimately experienced some of the highest investment returns in history. By the time those high returns occurred, they didn't have enough of their portfolio left for it to matter much.

Based on the historical record, we see that more conservative portfolios seem to have offered only limited downside protection and to have severely constrained the upside. This may come as somewhat of a surprise to the investment community, because choosing a more conservative portfolio mix has generally been the method of choice for financial advisors to stabilize future investment returns, and, therefore, the amount that can be taken out of portfolios safely. If this hasn't been effective historically, then it raises the obvious question: Is there a better way to ensure a portfolio will be able to provide adequate income throughout retirement?

Back to the (Possible) Future

Before we consider that question, however, we should recognize the limitation of the historical record, which gives us only a single sequence of returns to evaluate. While we examine data beginning in every month from 1946–1999, these data are all based on the same historical sequence of returns, which will almost certainly not be exactly replicated with any likelihood in the future. Another, and perhaps superior, way to analyze withdrawal rates is to perform a Monte Carlo analysis of a large number of possible future sequences of investment returns and inflation, based on data from the past.

In our method of analysis, possible future returns are simulated by randomly drawing return and inflation data one month at a time, with replacement, from the historical monthly data (1946–1999) and using these draws to construct a large number of possible return sequences. This procedure overcomes the statistical problems of using overlapping multi-period returns. We note, however, that in this type of analysis, returns and inflation figures in different periods are implicitly assumed to be statistically independent of each other.

In the analysis that follows, we listed the asset class returns and inflation rates for each month in the historical record from January 1946 to January 2000, and also each month's change in the consumer price index (CPI). We then drew out, at random and with replacement, 10,000 sequences of up to 50 years of annual historical events (involving a total of six million random draws from the monthly historical data). We then measured, in various ways, how the various portfolios would have evolved over time if we had withdrawn 4.5 percent of each portfolio in the first year, and adjusted the withdrawal amount for inflation thereafter. This 4.5 percent figure is chosen based on the Bengen and Pye analyses and also on the results of Figure 1, in order to test what appears to be a somewhat aggressive, but potentially sustainable, withdrawal rate over a variety of historical periods and conditions.

Table 1 shows a variety of statistics for the four portfolio mixes under examination.⁶ The first column shows the actual composition of the portfolio; the reader will note that these are the same asset class percentages that were used to create Figure 1. The second column lists the number of years of portfolio liquidation; the range of retirement periods, in five-year increments, is from 20 years to 40.

TABLE 1

Inflation-Adjusted Summary Statistics for Wealth Based on Annual Withdrawals of 4.5 Percent

Portfolio	Year	Mean	Std. Dev.	Pr(<0)	5th	10th	50th
Conservative	20	0.35	0.18	0.9	0.08	0.13	0.33
20% Stock 50% Bonds 30% Cash	25	0.15	0.20	24.8	-0.14	-0.09	0.12
	30	-0.08	0.22	67.4	-0.39	-0.34	-0.10
	35	-0.32	0.25	90.1	-0.67	-0.61	-0.34
	40	-0.58	0.28	97.1	-0.99	-0.91	-0.60
Balanced	20	0.68	0.40	0.9	0.15	0.24	0.62
40% Stock 40% Bonds 20% Cash	25	0.57	0.49	8.1	-0.07	0.03	0.48
	30	0.42	0.59	23.7	-0.32	-0.21	0.32
	35	0.26	0.71	41.1	-0.63	-0.48	0.13
	40	0.06	0.85	55.4	-1.00	-0.81	-0.09
Growth	20	1.16	0.81	1.2	0.18	0.31	1.00
60% Stock 30% Bonds 10% Cash	25	1.23	1.09	5.8	-0.03	0.13	0.98
	30	1.31	1.44	12.6	-0.29	-0.09	0.98
	35	1.43	1.94	20.1	-0.62	-0.36	0.96
	40	1.58	2.57	26.8	-1.03	-0.70	0.95
Aggressive	20	2.03	1.70	1.7	0.21	0.42	1.60
85% Stock 15% Bonds 0% Cash	25	2.59	2.60	4.8	0.01	0.28	1.87
	30	3.39	3.87	8.4	-0.25	0.11	2.25
	35	4.55	5.97	11.8	-0.59	-0.12	2.78
	40	6.17	8.99	14.7	-1.03	-0.42	3.47

Notes: All statistics are inflation-adjusted multiples of the initial accumulation amount. Withdrawals are inflation-adjusted. Source: Authors' calculation based on Ibbotson data.

Notes for Table 1: Table 1 shows the results of a Monte Carlo simulation of 10,000 hypothetical asset return histories for each of four investment portfolios listed in Column 1. Each history is constructed by drawing, at random with replacement, 600 months of returns and inflation from the actual historical record from 1946–1999, using monthly data from Ibbotson.

The statistics shown describe the remaining wealth portfolio at each point in time (Column 2), assuming annual withdrawal of 4.5 percent of the initial accumulation amount each year, adjusted for inflation.

The numbers in Columns 3–4 and 6–8 are expressed in multiples of the initial accumulation amount (e.g., for the conservative portfolio, after 20 years, the table shows that the mean amount in the left portfolio is 35, or 35 percent of the initial accumulations).

Column 5 is in percentage points. "Mean" is the arithmetic average of the remaining wealth amounts at each point in time across the 10,000 simulations. "Std. Dev." is the standard deviation across the 10,000 simulations. "Pr(<0)" is the "failure rate," or percentage of simulations in which the portfolio had run out of money.

The columns labeled 5th, 10th, and 50th show the amount of wealth in the portfolio at the bottom 5th, 10th, and 50th percentile of the 10,000 simulations. Thus, the 50th percentile is the amount of wealth at the exact middle of the distribution of the 10,000 simulations: in half of the simulations, portfolios had more wealth than is indicated; in the other half, the portfolios had less wealth than is indicated in this column.

The third column shows the mean amount of the original portfolio that the hypothetical retiree has left after withdrawals are made over each time period—that is, the average amount of wealth remaining in the portfolio. In the case of the aggressive portfolio, the mean portfolio size of the 10,000 trials, after 40 years of liquidation in retirement, was more than six times the original portfolio.

The fourth column calculates the standard deviation of the means shown in column three. This is an indication of the range of results. Standard deviation is one measure of the broad up-and-down swings, as seen in Figure 1. If the mean portfolio value were 0.42 (as it is for the 30-year liquidation of the balanced portfolio in Table 1), and the standard deviation is 0.59, then it means that roughly two-thirds of the trials fall between -0.17 ($0.42 - 0.59$) and 1.01 ($0.42 + 0.59$).⁷ In some cases, this is a highly significant figure to watch, because it measures the certainty or

uncertainty of the mean figure. For example, before we take too much comfort from the fact that the mean portfolio value after 40 years of liquidation is 6.17 times the original value, it is helpful to notice, by looking at the standard deviation figure, that at least two-thirds of the trials resulted in terminal portfolio values defined by an exceptionally high range: -2.82 to 15.16. Clearly there was plenty of opportunity for failure.

How should the negative numbers on the table be interpreted? These are perhaps best thought of as measuring the extent to which a retiree would have to rely on family or other external sources of funding to maintain his or her lifestyle. In much of what follows, the discussion focuses mainly on the likelihood that a portfolio runs out of money. However, one should also remember that the relative size of shortfalls is important: A portfolio that has a modest probability of being just barely exhausted after 40 years of withdrawals may be perceived as very different from one that has a small chance of leaving retirees with a deficit of two or three times their initial wealth after 40 years.

The figures presented in the fifth column of the tables are likely to be of considerable interest to financial advisors because they show the "failure rate": the percentage of trials in which the portfolio ran out of money before various periods were concluded. For example, in the growth portfolio, the reader can turn to the 30-year line, read across to the right and see that at a 4.5 percent (inflation-adjusted) liquidation rate, 12.6 percent of the trials result in portfolio extinction before the 30 years were up. In the aggressive portfolio, only 8.4 percent of the trials run out of money before the end of 30 years, while 14.7 percent of the aggressive portfolio trials experience failure by the end of 40 years.

The last three columns show the fraction of the portfolio that remains at the 5th, 10th and 50th percentiles of all trials. The numbers in this section are multiples of the initial portfolio value (adjusted for inflation), so numbers larger than one indicate values above the starting value, and a number less than one indicates a smaller value. Positive numbers (blue) indicate that there was still at least some of the portfolio remaining; negative numbers (red) indicate that the portfolio has been exhausted.

The percentile values provide another way to assess the likelihood of particular outcomes. For example, if the 5th percentile figure is equal to one, then in 5 percent of the 10,000 trials, the portfolio had a value less than or equal to its starting value (implying, of course, that in the other 95 percent of the trials, the portfolio had an equal or higher value). Of particular note is the rightmost column. It shows the 50th percentile, or the median, of the distribution of portfolio values. This is the "halfway point" of the distribution of returns. As mentioned earlier, if this figure is negative, that means that at least half of the trials resulted in the portfolio running out of money before the end of the time period. A number greater than zero indicates that at least half of the portfolios were able to survive despite the yearly liquidation.

As the reader explores Table 1, we should sound a note of caution. The investment and inflation figures used to generate these results were taken from past historical events; there can be no assurance that unforeseen economic conditions will not change the parameters of investment experience going forward. With that caveat in mind, the reader can see that these simulations tend to confirm Bengen's and Pye's analyses. The aggressive portfolio fails in just 8.4 percent of the 30-year Monte Carlo trials, meaning that it was able to sustain a 4.5 percent distribution factor in 91.6 percent of the trials.

Table 1 expands the field of vision considerably by examining various time periods and portfolio mixes. We can see, for example, that the 4.5 percent liquidation rate begins to look increasingly aggressive as you move toward less stock-heavy portfolio mixes, and it looks nearly suicidal for the conservative portfolio, where the possibility of portfolio extinction before 30 years is more than 67 percent.

Interestingly enough, the pattern of Table 1 shows that for shorter retirement periods, the less volatile (less stock-oriented) portfolios offer better chances of survival than the more aggressive alternatives. After 20 years, the conservative and balanced portfolios have a failure rate of less than 1 percent, compared with the 1.2 percent and 1.7 percent failure rates of the growth and aggressive portfolios.

As people live longer into retirement, however, the more volatile asset mixes offer much higher probabilities that the portfolio will sustain the 4.5 percent inflation-adjusted withdrawal rate. Indeed, the aggressive portfolio will support a 40-year retirement period, at this liquidation rate, roughly 85 percent of the time. A look at column 6 of Table 1 shows that the fifth percentage of trials—which defines the gloomiest five percent of these hypothetical outcomes—are not dramatically different for the aggressive portfolio than the less stock-heavy portfolios.

However, the appearance of stability can be misleading for two reasons. First, as mentioned earlier, the projected future sequences/trials are derived from historical data, which incorporate the historical equity premium, which may not be replicated in the future. Second, even if the premium somehow were to remain stable into the future, a look at the standard deviations in column 4 shows that the more stock-heavy portfolios come with a risk of much lower, as well as much higher, potential outcomes.

Making Withdrawals Last

At this point, it seems reasonable to return to one of our original questions: Is there any way to make it more likely that a given withdrawal rate can be supported over a long period? If future returns are comparable with past ones, then raising the stock portion of the asset mixes appears to afford a higher liquidation factor over longer time periods. But significant areas of uncertainty still remain. The aggressive portfolio, for example, fails in more than an eighth of all trials over a 40-year projected time horizon. This is comparable to a 60-year-old living to age 100—which, given current trends in longevity, may be an increasingly common scenario for which professionals need to plan.

In addition, higher stock allocations significantly increase the variability in the portfolio's value over the retirement period. On the downside, this means increased risk of portfolio extinction at an earlier age. On the upside, one wonders whether, in the face of an enormous run-up in portfolio values, clients wouldn't be tempted to abandon what might seem at the time to be an unnecessarily austere liquidation rate. In both cases, the feasibility of the fixed withdrawal rate strategy is called into question.

Some clients may be willing to overlook or accept these risks. But even for those with the highest risk tolerance, at some point, we can no longer raise the percentage of the portfolio allocated to stocks. And for the more risk-averse clients, it may simply be too much to ask them to tolerate the volatility of an aggressive portfolio. In such situations, we need to find other options that may have different characteristics.

The Longevity Factor

Throughout the preceding discussion, we have not really addressed the issue of uncertainty regarding length of life in retirement, despite the fact that it's hard to overstate the importance of this source of uncertainty. To illustrate, suppose that 100 65-year-old financial planning clients, all ready to retire, all with the same investments, were faced with the withdrawal problem we have been discussing. Each could "conservatively" plan on living 30 years in retirement and then make withdrawals from their respective portfolios that were expected to stretch the money out exactly that long (perhaps based on an analysis similar to that above). This would work well for members of the group for whom this guess about the ultimate length of life turns out to be exactly right. But for those who live longer than they expect, this procedure would result in real poverty after the "planned for" period. Meanwhile, those who end up living only a short while in retirement will leave behind a large, unplanned estate.

One way to deal with this uncertainty is for retirees to pool their assets and, therefore, their longevity risk through immediate annuities, which can convert retirement assets into a guaranteed lifetime income stream. Introducing an immediate annuity to a retirement portfolio raises a number of questions, but a crucial one would seem to be: Does the purchase of an immediate annuity with some of a person's retirement assets have a beneficial or deleterious effect on the sustainability of these liquidation rates that we have already examined?

To investigate that question, the authors ran the same 10,000 investment/ inflation iterations on different time horizons for the same four asset mixes, and extended the format of Table 1 to include annuitizing 25 percent and 50 percent of the investment portfolio. The monthly income payments from the fixed life annuity are based on the assumptions that the annuity pool's assets will achieve an annual rate of return of seven percent and that mortality patterns among the annuitants will match a recent mortality table. Given these assumptions, for a 65-year-old individual, the annual annuity payments amount to 9.05 percent of the amount annuitized. While prevailing interest rates have declined in 2001 (making currently available annuities more costly), we believe that this assumption is a reasonable estimate of rates typically available from reputable, competitive annuity providers. For example, one well-known provider offers (as of February 21, 2001) a monthly fixed annuity for age-65 males with income of roughly 8.57 percent of annuitized assets annually. Assuming that the relationship between this company's products and those of other insurance companies has remained constant recently, we estimate that the average income payment that a male, age 65, could

obtain from a fixed annuity was roughly 9.36 percent as of December 2000.⁸ We note, however, that there is considerable variation in available annuity rates and it can pay to shop around. Also, we note that the annuity provider's fees are already reflected in the assumed total rate of return, so there is no separate adjustment for cost in our analysis.

Tables 2, 3, 4 and 5 show the results of all trials for conservative, balanced, growth and aggressive portfolios, respectively, with 0 percent, 25 percent and 50 percent of the initial portfolio used to obtain a fixed annuity. The most complicated part of the calculations is keeping the yearly amount that is taken out of the portfolio equal in constant dollars despite the two sources of income (the portfolio withdrawals and the annuity payments). Because we assumed that the annuity portion of the income flow does not change with inflation, we had to raise the amount withdrawn from the unannuitized portion of the portfolio over time in order to maintain a constant total amount of inflation-adjusted income. As time goes on and the cumulative effects of inflation are felt more strongly, a declining portion of income is coming from the annuity and a larger portion from the non-annuitized portion of the portfolio.

TABLE 2

**Inflation-Adjusted Statistics for Wealth
In Conservative Portfolio**

Assuming Portfolio Withdrawals + Any Annuity Income = 4.5% Annually

Annuitization Rate	Year	Mean	Std. Dev.	Pr(<0)	Percentile		
					5th	10th	50th
0%	20	0.35	0.18	0.9	0.08	0.13	0.33
	25	0.15	0.20	24.8	-0.14	-0.09	0.12
	30	-0.08	0.22	67.4	-0.39	-0.34	-0.10
	35	-0.32	0.25	90.1	-0.67	-0.61	-0.34
	40	-0.58	0.28	97.1	-0.99	-0.91	-0.60
25%	20	0.37	0.16	0.1	0.14	0.18	0.35
	25	0.22	0.18	8.8	-0.04	0.01	0.20
	30	0.04	0.20	46.7	-0.24	-0.19	0.02
	35	-0.16	0.22	78.6	-0.47	-0.41	-0.19
	40	-0.38	0.24	93.5	-0.73	-0.67	-0.41
50%	20	0.39	0.14	0.0	0.19	0.23	0.38
	25	0.29	0.16	1.3	0.06	0.10	0.27
	30	0.16	0.18	18.7	-0.10	-0.05	0.14
	35	-0.00	0.20	54.5	-0.29	-0.23	-0.02
	40	-0.19	0.22	81.9	-0.51	-0.45	-0.21

Note: Statistics are inflation-adjusted multiple/trust ratio of the initial accumulation amount. Annuitly AFR is 7 percent. Withdrawals/income are adjusted for inflation.
Source: Authors' calculations based on Ibbotson data.

TABLE 3

Inflation-Adjusted Statistics for Wealth In Balanced Portfolio

Assuming Portfolio Withdrawals + Any Annuity Income = 4.5% Annually

Annuitization Rate	Year	Mean	Std. Dev.	Pr(<0)	Percentile		
					5th	10th	50th
0%	20	0.68	0.40	0.9	0.15	0.24	0.62
	25	0.57	0.49	8.1	-0.07	0.03	0.48
	30	0.42	0.59	23.7	-0.32	-0.21	0.32
	35	0.26	0.71	41.1	-0.63	-0.48	0.13
	40	0.06	0.85	55.4	-1.00	-0.81	-0.09
25%	20	0.65	0.33	0.1	0.20	0.27	0.60
	25	0.58	0.41	3.4	0.04	0.11	0.51
	30	0.48	0.50	14.9	-0.16	-0.07	0.39
	35	0.35	0.60	30.3	-0.40	-0.29	0.24
	40	0.20	0.72	46.0	-0.70	-0.57	0.06
50%	20	0.62	0.27	0.0	0.25	0.31	0.58
	25	0.58	0.34	0.6	0.14	0.20	0.53
	30	0.53	0.42	5.5	-0.01	0.07	0.45
	35	0.44	0.51	17.4	-0.19	-0.10	0.35
	40	0.33	0.61	32.8	-0.42	-0.31	0.21

Note: Statistics are inflation-adjusted multiples/percent of the initial accumulation amount. Annuitly AR is 7 percent. Withdrawals/income are adjusted for inflation. Source: Authors' calculations based on historical data.

TABLE 4

Inflation-Adjusted Statistics for Wealth In Growth Portfolio

Assuming Portfolio Withdrawals + Any Annuity Income = 4.5% Annually

Annuitization Rate	Year	Mean	Std. Dev.	Pr(<0)	Percentile		
					5th	10th	50th
0%	20	1.16	0.81	1.2	0.18	0.31	1.00
	25	1.23	1.09	5.8	-0.03	0.13	0.98
	30	1.31	1.44	12.6	-0.29	-0.09	0.98
	35	1.43	1.94	20.1	-0.62	-0.36	0.96
	40	1.58	2.57	26.8	-1.03	-0.70	0.95
25%	20	1.04	0.65	0.4	0.24	0.35	0.91
	25	1.12	0.89	2.6	0.09	0.23	0.92
	30	1.23	1.20	7.8	-0.10	0.07	0.94
	35	1.35	1.59	14.3	-0.34	-0.14	0.96
	40	1.50	2.10	21.2	-0.66	-0.39	0.95
50%	20	0.93	0.51	0.0	0.29	0.38	0.83
	25	1.04	0.72	0.6	0.19	0.31	0.89
	30	1.16	0.97	3.3	0.06	0.20	0.93
	35	1.30	1.28	8.0	-0.11	0.06	0.97
	40	1.46	1.71	14.1	-0.34	-0.13	1.02

Note: Statistics are inflation-adjusted multiples/percent of the initial accumulation amount. Annuitly AR is 7 percent. Withdrawals/income are adjusted for inflation. Source: Authors' calculations based on historical data.

TABLE 5

Inflation-Adjusted Statistics for Wealth In Aggressive Portfolio

Assuming Portfolio Withdrawals + Any Annuity Income = 4.5% Annually

Annuitization Rate	Year	Mean	Std. Dev.	Pr(<0)	Percentile		
					5th	10th	50th
0%	20	2.03	1.70	1.7	0.21	0.42	1.60
	25	2.59	2.60	4.8	0.01	0.28	1.87
	30	3.39	3.87	8.4	-0.25	0.11	2.25
	35	4.55	5.97	11.8	-0.59	-0.12	2.78
	40	6.17	8.99	14.7	-1.03	-0.42	3.47
25%	20	1.77	1.41	0.5	0.29	0.45	1.43
	25	2.28	2.17	2.2	0.16	0.37	1.68
	30	3.02	3.31	5.4	-0.02	0.26	2.07
	35	4.02	4.96	8.2	-0.25	0.12	2.53
	40	5.44	7.39	10.9	-0.57	-0.07	3.17
50%	20	1.51	1.11	0.1	0.32	0.45	1.24
	25	1.97	1.71	0.9	0.26	0.43	1.50
	30	2.61	2.67	2.5	0.15	0.37	1.86
	35	3.49	4.13	5.0	0.00	0.29	2.28
	40	4.72	6.25	7.4	-0.20	0.17	2.89

Note: Statistics are inflation-adjusted multiples/fractions of the initial accumulation amount. Annuitly ARI is 7 percent. 19th draws/income are adjusted for inflation.

Source: Author's calculations based on 10,000 trials.

Description of Tables 2-5: See note to Table 1. Tables 2-5 add an immediate fixed life annuity to the analysis. In these simulations, withdrawals are taken from the unannuitized portfolio in each year only to the extent that annual annuity income in that scenario falls short of equalling 4.5 percent of the initial total amount, adjusted for inflation. The statistics in the table describe the remaining unannuitized portfolio in each year (Column 2), in multiples of initial total assets.

By looking at columns 3 and 4, the reader will quickly see that the presence of the annuity does, indeed, offer some reduction in the number of trials that result in portfolio extinction. For example, for the conservative portfolio with no immediate annuity, almost 25 percent of the trials lead to failure at a 4.5 percent withdrawal rate after 25 years. After 30 years, the failure rate is 67.4 percent. Yet if 25 percent of the portfolio is annuitized, those figures drop to 8.8 percent and 46.7 percent, respectively. They fall to 1.3 percent and 18.7 percent when 50 percent of the portfolio is annuitized.⁹

Similar increases in portfolio sustainability can be seen in the other portfolio. The balanced portfolio, with zero annuitization, fails in 23.7 percent of the trials after 30 years; with 25 percent and 50 percent of the portfolio annuitized, the 30-year failure rate falls to 14.9 percent and 5.5 percent, respectively. In the growth portfolio, the chances of failure fall from 12.6 percent over 30 years with zero annuitization to 7.8 percent with 25 percent annuitization to 3.3 percent with 50 percent annuitization.

Notice that the reduction in uncertainty for the aggressive portfolio is accompanied by relatively modest drops in the mean terminal wealth after 40 years, from 6.17 times the original portfolio (with no annuity) to 5.44 (with 25 percent annuitized), to 4.72 times the original portfolio when half the portfolio is annuitized. There are, however, substantial drops in the standard deviation of terminal wealth as well: from 14.7 in the unannuitized case, to 10.9 with 25 percent annuitized, to 7.4 with 50 percent annuitized. In fact, while mean terminal wealth after 40 years falls by 24 percent (comparing the unannuitized case with the 50 percent annuitized case), the standard deviation of ending wealth falls by nearly 50 percent. Effectively, the retiree is trading the possibility of extremely large accumulated wealth levels for a greater degree of certainty that the portfolio can generate income throughout his or her lifetime. This leads to an interesting observation: By providing a stable source of income, the fixed life annuity can effectively substitute for the cash or bond part of a retiree's portfolio. This may enable even very risk-averse retirees feel more secure about

investing their unannuitized assets more aggressively.

Comparing the Historical and Monte Carlo Approaches

Table 6 compares the results of the historical and Monte Carlo approaches to examining the sustainability of a 4.5 percent withdrawal rate.¹⁰

TABLE 6					
What Is The Probability of Running Out of Money With a 4.5 Percent "Total Withdrawal" Rate?					
Portfolio	Retirement Period	Historical	Monte Carlo		
		No Annuity	No Annuity	25% Annuitized	50% Annuitized
Conservative	20 Years	1.2%	0.9%	0.1%	0.0%
20% Stock 50% Bonds 30% Cash	25 Years	62.3%	24.8%	8.8%	1.3%
	30 Years	100.0%	67.4%	46.7%	18.7%
	35 Years	100.0%	90.1%	78.6%	54.5%
	40 Years	n/a	97.1%	93.5%	93.8%
Balanced	20 Years	0.2%	0.9%	0.1%	0.0%
40% Stock 40% Bonds 20% Cash	25 Years	35.7%	8.1%	3.4%	0.6%
	30 Years	64.8%	23.7%	14.9%	5.5%
	35 Years	62.2%	41.1%	30.3%	17.4%
	40 Years	n/a	55.4%	46.0%	32.8%
Growth	20 Years	1.7%	1.2%	0.4%	0.0%
60% Stock 30% Bonds 10% Cash	25 Years	29.1%	5.8%	2.6%	0.6%
	30 Years	43.8%	12.6%	7.8%	3.3%
	35 Years	47.0%	20.1%	14.3%	8.0%
	40 Years	n/a	26.8%	21.2%	14.1%
Aggressive	20 Years	7.6%	1.7%	0.5%	0.1%
85% Stock 15% Bonds 0% Cash	25 Years	25.7%	4.8%	2.2%	0.9%
	30 Years	33.8%	8.4%	5.4%	2.5%
	35 Years	24.8%	11.8%	8.2%	5.0%
	40 Years	n/a	14.7%	10.9%	7.4%

Note: Annuity payments = portfolio withdrawal = 4.5 percent of initial accumulation, adjusted for inflation.

Description of Table 6: Table 6 compares the results of the historical analysis of withdrawal rates with the monte-carlo analysis. Results are shown for the four portfolios in column 1, and withdrawal periods in column 2. The "Historical" column shows the fraction of rolling historical periods in which withdrawing 4.5 percent of the initial portfolio amount (adjusting for inflation with each withdrawal) would have exhausted the portfolio before the end of the period. The "Monte Carlo" columns show the fraction of simulations (in cases of 0 percent, 25 percent and 50 percent annuitization) in which the unannuitized portfolio was exhausted before the end of the retirement period. Both the historical and Monte Carlo approaches reflect a 1 percent asset management fee. Historical analysis for 40-year period was not conducted; there were not enough rolling 40-year periods for a meaningful analysis.

At first glance, the results of the two analyses seem very different: For all of the portfolios and at all horizons, the historical analysis suggests that the failure rates are much higher than the Monte Carlo analysis in which there is no annuity. Here it must be remembered that the historical analysis is based on a unique pattern of events, in which the bear market of the mid-1970s and the high inflation of the late 1970s and early 1980s figure prominently. Whether these events will ever occur in the same way in the future is an open question; the Monte Carlo analysis reduces the

importance of this historical pattern. Note also that in the historical analysis, the failure rate for the aggressive portfolio is higher over the 30-year periods in the sample than it is over the 35-year periods. This result demonstrates an important weakness of the historical analysis: there are more rolling 30-year periods over the span 1946–1999 than 35-year periods. Thus, the "sample" of 35-year periods does not include those periods beginning in the late 1960s, while the "sample" of 30-year periods does. Given the market performance in the early to mid-1970s, the result in the table is understandable. This again serves to highlight the important advantage of the Monte Carlo analysis: the Monte Carlo examination of the failure rates is based on the same number of independent trials for each time period.

Despite the difference in the absolute level of the "failure rates" in the two analyses, both point to one conclusion: Portfolios with a significant stock market exposure had much higher probabilities of sustaining systematic withdrawals longer into retirement. In addition, the table clearly shows the significant improvement in the "failure rate" that the annuity provides: For all time periods and for all portfolios, the addition of the annuity leads to a decline in the portfolio failure rates.

Discussion

It appears that Bengen and Pye were substantially correct in their analysis of portfolio sustainability. For 30-year retirement periods, a 4.5 percent withdrawal rate succeeds more than 90 percent of the time only if the asset mix is very heavily weighted toward stocks. Retirees who select a more conservative (less stock-heavy) retirement mix might be able to achieve slightly more consistency if they expect a short retirement, but the probability of failure will be dramatically higher over time periods of 20 years or longer.

Our analysis also shows that the 4.5 percent withdrawal factor can be sustained with more certainty, for longer time periods, by adding the risk-pooling characteristics of an immediate annuity to the overall retirement portfolio. In much the same way as asset classes are used to hedge financial risks, immediate annuities can be used to hedge longevity risk. It is important to note, however, that the pooling of longevity risk has consequences other than portfolio withdrawal stability. If the retiree dies early in the retirement years, after having purchased an immediate annuity, then the investment in the pool continues to pay other annuity holders rather than the retiree's heirs. This opportunity cost should be assessed before any decision is reached regarding the structure of retirement portfolios.¹¹

In addition, the simple fact that the annuity dampens future uncertainty virtually guarantees that there will be less opportunity for the retiree portfolios to grow as large as in the unannuitized case. The presence of the pooling arrangement effectively compresses the tails of the distribution of future portfolio wealth—making it less likely that the investor will go broke, but also reducing the size of the accumulation that will be passed to heirs.

Given the trend toward increased longevity and the possibility of major medical advances in the future, pooling the risk of longevity offers potential benefits that have not been captured by the tables presented here. While the portfolios that contained no annuities actually ran out of money in the years indicated in tables 1–5, the retirement portfolios that were 25 percent or 50 percent invested in annuities were still making income payments to the hypothetical retirees 40 years after they left work. Even 50 or more years after retirement, the annuities would still be generating income for life, albeit on a noninflation-adjusted basis.

Finally, we note that there are at least two important ways in which further analysis could extend the initial results presented here. First, we have not attempted to model the impact of taxes on the analysis. In some circumstances, the effect of taxes is a straightforward extension of our results. For example, if a client's retirement assets were held entirely in qualified retirement plans, income received would just be lower by a percentage equal to the income tax rate.¹² But if assets are held in nonqualified vehicles, the effects of both income taxes and capital gains taxes need to be considered. The interaction of these taxes may have important implications for both the level and sustainability of retirement income, optimal portfolio choices and the use of immediate and deferred annuities. It would be useful to examine these effects in much greater detail.

Second, when discussing annuities in this article, we have focused exclusively on immediate annuities that pay a guaranteed, *fixed* level of income (noninflation-adjusted) for the life of the annuitant(s). While this type of fixed-income annuity may be what immediately comes to mind when one hears the word "annuity," other types of income-generating life annuities are currently available to retirees. In particular, immediate *variable* annuities provide income

payments for life that periodically change (either up or down) to reflect the realized recent performance of whatever assets are held in the annuity's asset pool.¹³ Variable immediate annuities based on stock market investments therefore allow annuitants to hedge their longevity risk without giving up exposure to the investment risks and returns of the stock market. Further analysis should certainly evaluate the usefulness of this type of immediate annuity as yet another tool to help retirees maximize their portfolio income.

The debate over how best to determine, maximize and stabilize the amount of a portfolio that can be liquidated in retirement will certainly lead professional retirement planners toward a better understanding of the immediate annuity as investment variable. This, in turn, should lead to better recommendations, decisions and, perhaps, more prosperous retirements for the clients of financial advisors who will have to live with the many uncertainties of the future.

Endnotes

1. William P. Bengen, "Determining Withdrawal Rates Using Historical Data," *Journal of Financial Planning*, [January 1994](#), pp. 14–24.
2. Throughout the Bengen analyses, inflation rates were measured using changes in the consumer price index.
3. William P. Bengen, "Conserving Client Portfolios During Retirement, Part III," *Journal of Financial Planning*, [December 1997](#), pp. 84–97.
4. See Figure 4 in Gordon B. Pye's "Sustainable Investment Withdrawals," *Journal of Portfolio Management*, Summer 2000, pp. 73–83. Pye has subsequently extended his analysis to incorporate the effects of management expenses and taxation on sustainable withdrawals; see "Adjusting Withdrawal Rates for Taxes and Expenses," *Journal of Financial Planning*, [April 2001](#), pp. 126–136.
5. Here, and in all subsequent analyses, no tax effects are calculated.
6. Note that in all analyses, we assumed that the portfolio has an annual expense of one percent a year.
7. The portfolio return data are almost certainly not normally distributed, so two-thirds is, at best, a rough approximation of variation around the mean.
8. This estimate is based on the average annuity rate (for 50 insurance companies) as reported in *Comparative Annuity Reports*, Vol. 21, Issue 12, December 2000. This publication reports that the average SPIA rate for a male, age 70, with a 10-year certain period, is \$8.19 a month per thousand dollars annuitized. On an annual basis, this is equivalent to 9.828 percent of the annuitized amount.
9. The extinction percentages for these portfolios were calculated as of the time when it was no longer possible to take income equivalent to 4.5 percent (inflation-adjusted) of the original retirement portfolio, even though some income was being paid continuously.
10. We did not conduct the historical analysis for 40-year periods; we felt there were not enough rolling 40-year periods for a meaningful analysis.
11. Retirees who are nervous about putting their assets into the risk pool can elect for a period-certain payout of 10 or 20 years. This effectively removes a portion of the annuity's value from the risk pool and addresses risk in the most extreme cases of premature death. Because this option removes assets from the pool, it also generates somewhat lower payment amounts than the purchase of a "straight" life annuity without the period-certain feature, especially at older ages.
12. Of course, this also assumes that a single income tax rate applies and is constant over time.
13. In their most basic form, these annuities operate by guaranteeing that some set share of the assets in the annuity pool will be distributed to each annuitant for life. However, because the value of the investments in the pool fluctuates, the value of the annuity payments (which are proportional to each annuitant's share in the pool) also fluctuate. As a result, the annuitant hedges longevity risk (they will always get a payment), but not investment risk (the payment may go up or down, depending on how the assets perform).

Acknowledgments

We thank the members of the TIAA-CREF Institute Financial Advisor Advisory Board, especially Janet Briaud; attendees at a session of the FPA conference in September 2000; and three anonymous referees, for valuable comments and discussions. The opinions expressed in this document are those of the authors alone, and not

necessarily those of TIAA-CREF. The authors are solely responsible for any errors.