Guidelines for Withdrawal Rates and Portfolio Safety During Retirement

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Executive Summary

- The existing literature for retirement portfolio withdrawal rates suggests that a real withdrawal rate of 4 percent of the initial portfolio is "safe." But this paper demonstrates that a blanket "4 percent withdrawal" rule may be an oversimplification of a complex set of circumstances.
- Risk tolerance, asset allocation, withdrawal size, and expected returns all affect the process of withdrawing from a retirement portfolio. To advance previous research, this paper uses 21 stock/bond allocations and 71 withdrawal rates, for 1,491 possible combinations. For each of these combinations, 10,000 bootstrap iterations are run for 30-year periods.
- Results show that withdrawals rates as high as 5.5 to 6 percent can be achieved, but only at a 25 to 30 percent chance of running out of money and with stock allocations 75 to 100 percent. A 4.4 percent withdrawal rate with a 50/50 bond/stock allocation has a 10 percent chance of running out of money.
- To visually illustrate the results for clients, the paper develops easy-to-understand withdrawal contours, runout contours, and balance remaining contours that clearly reveal the relationship between asset allocation, withdrawal rates, the chance of running out of money, and estate building. First, given a tolerance for the chance of running out of money, the largest amount that can be withdrawn can be determined. Second, the contours can be used to provide the client's optimal asset allocation for a fixed withdrawal rate and a given tolerance for running out of money. Third, the withdrawal amount at various levels of tolerance for running out of money can be determined while holding the asset mix constant.

Optimal asset allocation strategies and optimal withdrawal strategies are of significant importance to individual investors. The determination of an optimal asset allocation and its optimal withdrawal strategy is complex and difficult, however, because two interrelated decisions must be made: selecting an appropriate asset allocation and choosing a suitable withdrawal rate. Adding to the complexity is that these choices often are the results of mutual decisions made by more than one party, frequently an individual investor and the investment advisor.

A large body of research on "safe" withdrawal rates for individuals has determined that a real withdrawal rate in the neighborhood of 4 percent of the initial retirement portfolio has a "low" chance of running out of money. Several studies cite different "safe" rates of withdrawal that change with the asset mix. One of the most common types of investigation into withdrawal rates looks at the number of years a portfolio of a given size will endure when withdrawing

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a fixed amount (sometimes inflation adjusted). These studies often use rolling historical periods, such as 30-year periods from 1951 to 1980, 1952 to 1981, and so on. If the results indicate that all portfolios lasted at least 30 years for withdrawal rates of 4 percent, these studies conclude that a 4 percent withdrawal rate is "sustainable."

While a specific withdrawal rate might be safe (or sustainable) under certain circumstances, a complete explanation should specify what is meant by "safe." Upon closer examination, the previous findings are not as robust as they initially appear. First, the rolling historical period approach looks at a very small number of 30-year periods, limited by the available historical record. Second, even though the small sample shows up with a very small number of failures when using the 4 percent strategy, the proportion of failures as a measure of probability is a poor metric, given the small sample size. Relative frequency is a good measure of probability only for large samples.

An optimal withdrawal strategy must address two factors: (1) asset allocation (which affects portfolio risk and rate of return) and (2) the withdrawal rate or withdrawal amount. Ideally, these two factors should be examined over a sample space that is sufficiently large to include all possible reasonable combinations of asset allocations and withdrawal rates. Many researchers have sought to identify an appropriate strategy for several asset allocation-withdrawal rate pairs, but to date there has been little research to examine these factors simultaneously and extensively across a wide range of values for each factor.

This paper uses a bootstrap algorithm in which re-sampling with replacement occurs. This methodology provides a more extensive examination, using thousands of 30-year periods, many different withdrawal rates, and many asset allocations. It also generates more robust measures of the risk of running out of money and therefore more robust determinations of "safe" withdrawal amounts under varying conditions. Since this technique examines many more combinations of asset allocation and withdrawal strategies than previous studies, the results should help inform decisions on optimal asset allocation and optimal withdrawal strategies. The following section provides a review of the relevant literature.

Literature Review

Limited attention has been focused on the individual investor and the withdrawal phase of the life-cycle approach (Modigliani and Brumberg 1954). Bengen (1994, 1996, 1997, 2001, 2004, and 2006) has been at the forefront of the safe-withdrawal investigation and has provided perhaps the most extensive illustrations on the topic of withdrawals during the retirement phase of the life cycle. Generally, he has shown that 4 percent of the initial portfolio is a "safe" withdrawal rate with a 50 percent stock allocation. Kwok, Milevsky, and Robinson (1994) and Milevsky, Kwok, and Robinson (1997) use Canadian mortality tables and asset class returns to show that an optimal asset allocation during retirement is 75 to 100 percent in equities. Guyton—(2004) using multiple mutual funds, a sophisticated withdrawal scheme, and a selected time frame of 1973–2003—shows that a portfolio subject to his set of decision rules can produce a withdrawal rate that "ranges from 5.8 percent to 6.2 percent depending on the percentage of the portfolio that is allocated to equity classes." In the absence of a description of the methodology, it is virtually impossible to replicate Guyton's results.

Using historical rates of returns on asset classes, Cooley, Hubbard, and Walz (1999) demonstrate that a portfolio invested 75 percent in large-cap U.S. stocks and 25 percent in intermediate-term, high-grade corporate bonds can be subject to an inflation-adjusted sustainable withdrawal rate of 4 to 5 percent. Cooley et al. (2003) find a real withdrawal rate of 4+ percent sustainable with a 75 percent chance of success included in the definition of sustainable. Unfortunately, they do not provide a rational basis for regarding a 25 percent chance of running out of money as "safe." What one investor considers "safe" might be viewed as extremely risky by a different investor. Tezel (2004), using Monte Carlo simulation methods, illustrates that when the chance of running out of money is below 8 percent, annual (inflation-adjusted) withdrawals of 4.5, 5.5, and 6.5 percent are possible for time horizons of 30, 20, and 10 years respectively. Ragsdale, Seila, and Little (1994) provide a mathematical algorithm that uses discounted cash flows to determine the optimal withdrawal rate from tax-deferred retirement portfolios. In a new approach to the topic, Milevsky and Robinson (2005) introduce the concept of a stochastic present value, which addresses the withdrawals issue from an actuarial perspective.

In a recent paper, Dus, Maurer, and Mitchell (2005) compare and contrast several types of phased withdrawal strategies. Dus et al. (p. 183) caution that "there is no clearly dominant strategy, because all involve trade-offs between risk, benefit, and bequest measures, and individual preferences may vary." Dus et al. have an excellent review of the literature concerning withdrawal strategies for anyone requiring a more complete and wider selection than provided here.

If attention is confined to real withdrawals from a portfolio over 30 years, the literature appears to contain conflicting results. Withdrawal rates considered safe or sustainable vary from 3 percent to more than 6 percent, while optimal asset allocations range from 50 percent to 100 percent stock. The results are all plausible because the outcome depends on the subjective definition of *sustainable* and *safe*. Further clouding the analysis is the composition of the portfolio (different types of domestic stocks, Treasury inflation-protected securities, international stocks), whether the data are monthly, quarterly, or annual, whether the study uses rolling historical periods, a bootstrap, or Monte Carlo methods. This paper addresses the following question: When withdrawing a constant real amount, what is the probability of running out of money before 30 years have elapsed over a broad range of withdrawal rates and asset allocations between stocks and bonds?

Data and Methodology

Imagine there is a retirement portfolio that the client expects will last for the next 30 years. The value of the portfolio will change each year depending on how the inflation-adjusted (real) rates of return for stocks and bonds affect the portfolio balance. At the end of the year, a fixed amount is withdrawn. The process is repeated until either 30 years have passed or the portfolio runs out of money. The study counts the number of times out of 10,000 that each 30-year simulation runs out of money. To be complete, the simulation must be done for both a large range of withdrawal amounts and large range of stock/bond allocations.

There Are 21 Stock/Bond Allocations

Only stocks and bonds are assumed to be in the portfolio. Let λ represent the proportion of the portfolio devoted to stocks. Bonds then are $(1 - \lambda)$ of the portfolio. can take on values of 0, 0.05, 0.10,…, 0.95, and 1.0. There are 21 distinct stock/bond allocations: 0 percent stocks/100 percent bonds, 5 percent stocks/95 percent bonds,…, 95 percent stocks/5 percent bonds, 100 percent stocks/0 percent bonds.

Withdrawal Strategies

Withdrawals are a fixed percentage of the starting balance of the portfolio. The withdrawal amounts begin at 2 percent and extend to 9 percent in increments of 0.1 percent. This results in a total of 71 withdrawal rates.

Combining Asset Allocation and Withdrawal Strategies

From the information above, there are 21 asset allocation strategies and 71 withdrawal rates, resulting in (21 asset allocations × 71 withdrawal rates) = 1,491 possible combinations of allocations and withdrawal rates. For *each* of these conditions, the percentage of times (out of 10,000) that the portfolio runs out of money before 30 years have passed is calculated. The smaller this percentage, the more successful ("safer") the withdrawal strategy.

Data, Variables, and Notation

Annual real (inflation-adjusted) rates of return from 1926 through 2005 for stocks (S&P 500) and bonds (intermediate-term U.S. Treasury bonds) are obtained from *Stocks, Bonds, Bills and Inflation 2006 Yearbook*, Ibbotson Associates. Some authors used nominal rates of return and then adjusted the withdrawals each year for inflation such that the withdrawal amount was the same amount in real terms. We chose to use real dollars throughout and avoid the annual inflation adjustment. The outcomes of either process should be the same irrespective of where the adjustment for inflation is made, whether in the withdrawal rate or in the rate of return earned by the investment. Taxes and transaction fees are omitted throughout.

The following variables are used to define the model and to describe the estimation process:

- t = the year in which the withdrawal occurs; $t = 1, 2, \ldots T$
- P_o = \$100, the starting amount of the portfolio, beginning in year 0
- w = the withdrawal amount at the end of each year (71 different rates)
- r_{st} = annual real (inflation-adjusted) rate of return on stocks at period t
- r_{bt} = annual real (inflation-adjusted) rate of return on bonds at period t
- λ = the proportion of the portfolio designated for stocks (21 different allocations)

Allocations are made only between stocks and bonds; thus, $(1 - \lambda)$ is the proportion of the portfolio allocated to bonds. For example, a λ of 0.30 means that 30 percent of the portfolio is allocated to stocks and 70 percent is allocated to bonds. Rates of return, r_{st}, and r_{bt} vary with t.

$$S_o = \lambda P_o \tag{1}$$

is the starting amount of the portfolio allocated to stocks.

$$B_o = (1 - \lambda)P_o \tag{2}$$

is the starting amount of the portfolio allocated to bonds. For each year $t = 1, 2, \ldots T$,

$$S_t^* = S_t(1 + r_{st}) \tag{3}$$

is the value of the stock portfolio before rebalancing at end of year t

$$B_t^* = B_t(1 + r_{bt}) \tag{4}$$

is the value of the bond portfolio before rebalancing at the end of year t.

At the end of the year, the withdrawal is removed from the combined portfolio and then the portfolio is rebalanced back to its original proportions. Thus, the end-of-year value of the portfolio is given by

$$P_t^* = (S_t^* + B_t^*) - w. \tag{5}$$

If P_t^* is negative, it is set equal to zero. The beginning-of-the-year stock and bond amounts are given by $S_{t+1} = \lambda P_t^*$ and $B_{t+1} = (1 - \lambda)P_t^*$, respectively.

The Bootstrap Algorithm

As noted earlier, there are 21 different asset allocations and 71 different withdrawal rates for a total of 1,491 asset allocation/withdrawal combinations. The following steps are repeated 10,000 times[1] for each of these 1,491 combinations.

Set $t = 1$, $P_0 = \$100$, $T = 30$. Select the starting value of w, the withdrawal amount.

(a) Randomly generate a number between 1926 and 2005 (inclusive), which is the "current year" subscript t. Obtain rb and rs for this "year." (This retains the asset class cross-correlations.)

(b) Compute P_t^* of equation (5)

(c) Increment t by 1. If $t > T$, save the value of P_t^* for analysis, otherwise, go to step (a).

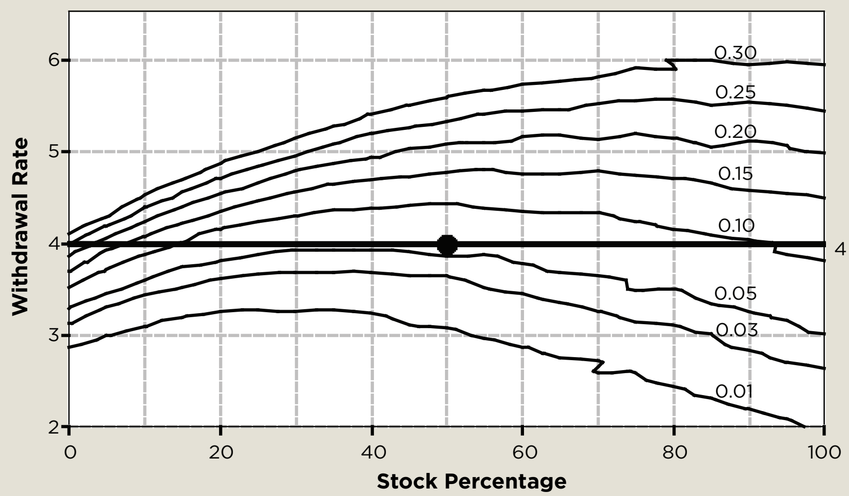
The steps above constitute a single iteration. There will be 10,000 such iterations for each of the conditions.

Results

The study generates a plethora of data. Rather than attempt to present the data tabularly, two graphs have been developed that capture the essence of the study. The 21 × 71 grid of probabilities of running out of money can be plotted into a three-dimensional surface map. Different slices of the surface map provide what will be referred to as runout contours and withdrawal contours. These contours offer visual "maps" for examining the trade-offs between asset allocation strategies, withdrawal strategies, and portfolio safety. (Withdrawal rates of 7 percent or more invariably resulted in unacceptable runout rates; these results are not shown on any of the contour maps.)

Figure 1: Runout Contours by Withdrawal Rate and Stock Percentage

The number on each contour line indicates the probability of running out of money as withdrawal rates and stock percentages change.



• marks a 4 percent withdrawal rate with 50 percent stocks and 50 percent bonds.

Runout Contours: Constant Probability of Running Out of Money

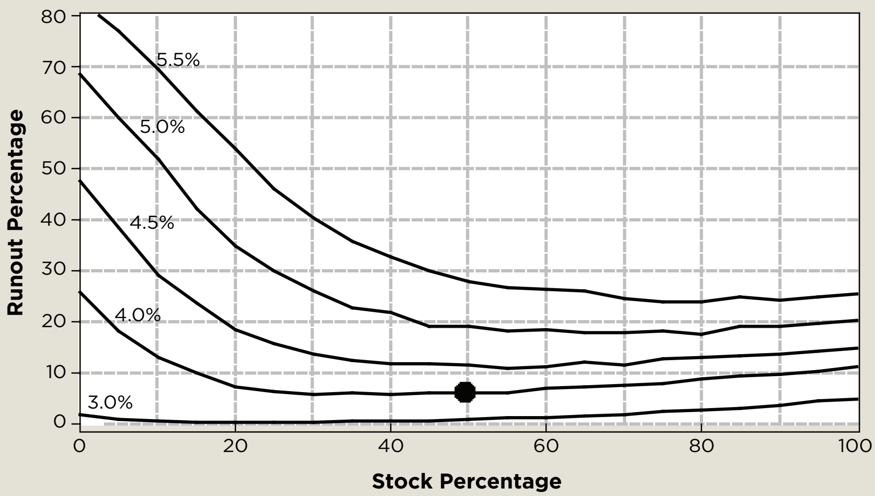
Runout contours in Figure 1 show all possible combinations of withdrawal rates and stock percentages that provide the same probability of running out of money before the 30 years have elapsed. For example, a 10 percent chance of running out of money can be attained by a 3.5 percent withdrawal rate with a portfolio of 0 percent stock, or also attained by a 4.4 percent withdrawal rate with 50 percent stock. In other words, if an individual investor is willing to accept a 10 percent chance of running out of money, the maximum withdrawal rate is about 4.4 percent given a 50 percent stock allocation. Similarly, at a 15 percent runout probability, the highest withdrawal rate is 4.7 percent given an allocation of about 57 percent stocks. Since the contours do not intersect, it is clear that higher withdrawal rates result in higher runout rates, given any asset allocation. Less trivially, the relationship between withdrawal rates and asset allocation shows that if one is willing to take high risk of running out of money, say 25 to 30 percent, very high stock percentages (80 to 100 percent) in the portfolio will support withdrawal rates of 5.5 to 6 percent.

The horizontal line drawn at 4 percent captures much of what appears in the literature, since this withdrawal rate is the one most often claimed to be safe. Notice that the 4 percent withdrawal rate cuts through several runout contours. A 4 percent withdrawal rate can obtain a 20 percent runout rate at very high concentrations of bonds (about 97 percent bonds) with a generally falling runout rate as the percentage of stocks increases. At 50 percent stocks, the 4 percent withdrawal rate falls between the 5 percent and 10 percent runout contours, indicating slightly less than a 6 percent chance of running out of money. The 4 percent withdrawal rate cuts through the 10 percent runout rate twice, once at about 17 percent stocks and once again at 92 percent stocks.

The runout contour shown in Figure 1 can be very helpful in illustrating the trade-off involved between the size of the annual withdrawal, the risk of running out of money, and the optimal asset allocation. For example, a client might ask, "If I am willing to take a 20 percent chance of running out of money, what is the largest withdrawal amount that I can take?" The answer is easily read from Figure 1: about 5.2 percent with 60–80 percent in stocks. If the investor is willing to take only a 10 percent chance, the amount falls to about 4.4 percent with about 50 percent stock. At a 5 percent chance, the largest withdrawal is almost 4 percent with 30–40 percent in stock. At a 3 percent risk of running out of money, 3.75 percent can be withdrawn when stocks constitute 35–45 percent of the portfolio. With increasing withdrawal rates comes an increasing likelihood of running out of money. But that likelihood is minimized by choosing the best asset allocation, which tends to become more

Figure 2: Withdrawal Contours by Runout Percentage and Stock Percentage

The number on each withdrawal contour is the withdrawal rate.



• marks a 4 percent withdrawal rate with 50 percent stocks and 50 percent bonds.

concentrated in stocks at higher withdrawal amounts.

Withdrawal Contours: Constant Withdrawal Rates

Withdrawal contours in Figure 2 are a view of the same data, but from a different perspective. This view shows how the probability of runouts varies with changes in the asset allocation for constant withdrawal amounts of 3, 4, 4.5, 5, and 5.5 percent. Looking at the 4 percent withdrawal contour, one can observe that the probability of running out of money is approximately 10 percent when stocks make up 16 percent of the portfolio. The probability of running out of money falls to about 5.5 percent when stocks make up between 30 percent and 50 percent of the portfolio. The probability of a runout on the 4 percent withdrawal contour increases slightly for stock percentages beyond 60 percent.

It is apparent that there are relationships between these three variables. First, since the contours never cross, it is clear (but trivial) that for a given stock/bond allocation, the lower the withdrawal rate is, the lower the runout risk will be. Second, for any constant withdrawal rate (withdrawal contour), very high concentrations of bonds increase the risk of runout. Third, in no circumstance was 100 percent stocks advantageous; for every contour, a lower risk of runout can be achieved at some asset allocation less than 100 percent stocks. Fourth, the minimums on each contour tend to require increasing amounts of stock as the withdrawal rate increases. For example, the lowest runout risk (near 0 percent) for the 3 percent withdrawal rate occurs in the 15–35 percent stock percentage. The lowest risk of runout for a 4 percent withdrawal is about 30–50 percent stocks, while it is about 55–80 percent and 75–90 percent stocks respectively for 5 percent and 5.5 percent withdrawal rates.

Clearly, asset allocation is important in minimizing runout risk for a constant withdrawal rate. Bonds play a valuable role in obtaining the best trade-off between runouts and withdrawal rates, but the withdrawal contours demonstrate that having more than 70 percent in bonds (less than 30 percent stocks) increases runout probability for all withdrawal rates of 4 percent or more.

Bengen's (1994) results indicated that the optimal asset allocation was about 50/50. The results in Figure 1 tend to confirm his conclusions for withdrawals in the 4–4.5 percent range. Contrary to Bengen's (1994) results, a 3 percent withdrawal rate is not "absolutely" safe. There is a small (slightly less than 1 percent) but real, chance for running out of money at the optimal 50 percent allocation.

On both Figures 1 and 2, a solid black circle has been inserted to pinpoint the Bengen (1994) outcome with 50/50 stock bond allocations with a 4 percent real withdrawal rate. Contrary to Bengen's findings that such a portfolio always lasts more than 30 years, these results show slightly less than a 6 percent probability of running out of money. The Cooley et al. (1998) outcomes are difficult to pinpoint on the figures since Cooley et al. used different data (long-term high-grade corporate bonds) and consequently obtained different results.

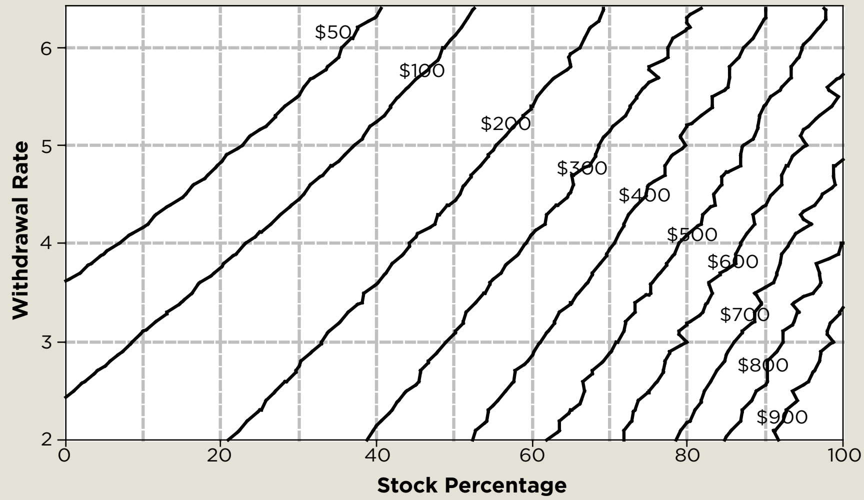
Balance-Remaining Contours: Constant Balance Remaining

There may be money left in the portfolio at the end of 30 years. In fact, if the market had a good run early in the 30-year period, there may be a sizeable amount remaining. The amount withdrawn each year and asset allocation will interact in determining how much money remains.

The balance-remaining contours presented in Figure 3 illustrate the effect on Balance Remaining[2] of different withdrawal rates and stock percentages. This analysis might be useful when clients desire to leave an inheritance in addition to sustaining withdrawals from a portfolio. First, the balance-remaining contours all slope upward to the right. Moving upward on the $50 contour implies that the balance of $50 remaining can be maintained at a 4 percent withdrawal with 7 percent stock, or a 5 percent withdrawal with 22 percent stock, or a 6 percent withdrawal with 35 percent stock. For all the balance-remaining contours, increasing the withdrawal rate requires increasing the stock percentage in order to remain on the contour. Second, for any withdrawal rate, the average balance remaining tends to increase as the percentage of stock increases. For example, at a 4 percent withdrawal rate, 22 percent stock obtains a $100 ending balance; 43 percent stock results in a $200 ending balance, while 79 percent stock results in a $500 balance at the 4 percent withdrawal rate. Two reminders may be in order: (1) The starting portfolio was $100, so an ending portfolio of $500 indicates that the portfolio not only survived 30 years of withdrawals, it thrived. (2) The average balance remaining is a measure of successful portfolios; a large percentage of portfolios have no money left before 30 years elapse. The footnote should serve as a cautionary note that average balance remaining is unlikely to occur in practice.

Figure 3: Balance Remaining Contours by Withdrawal Rate and Stock Percentage

Each line shows asset allocation and withdrawal combinations that leave the same balance remaining.



Numbers on the contours are the balance remaining ($) after 30 year of withdrawals from the $100 initial portfolio.

Conclusions

This study used a bootstrap simulation to calculate the probabilities of running out of money in a retirement portfolio when annual withdrawals of a constant real amount are carried out. Expanding on previous research in this area, the study incorporates many more asset allocations and withdrawal rates. There were 71 withdrawal rates ranging from 2 percent to 9 percent in 0.1 percent increments and 21 different stock/bond allocations ranging from 0 percent stocks to 100 percent stocks in 5 percent increments. Ten thousand 30-year sequences were generated for each of the 1,491 combinations of withdrawal and stock allocations. Since the results are more robust, they can be interpreted with a greater degree of confidence. This study adds to the growing body of work in several ways:

1. It demonstrates why an absolute withdrawal rate rule (of 4 percent, or three percent, or five percent), without reference to asset allocation and tolerance of the investor to the probability of running out of money, may be inappropriate.
2. It develops withdrawal contours and runout contours that depict the relationship among withdrawal rates, asset allocations, and the probabilities of running out of money. These contours provide financial advisors the visual ability to place clients on these contours as a function of asset allocation, tolerance for the chance of running out of money, and acceptable asset allocation. The contours also enable clients to better understand and interpret the advice and suggestions of financial advisors.
3. The withdrawal contours and runout contours make more straightforward and robust the understanding of interrelated decisions involving asset allocation and withdrawal rate strategies. The adage "a picture is worth a thousand words" applies to these contours. They can be used by clients and investment advisors to discuss multiple asset allocation/withdrawal rate scenarios in a simple, yet accurate manner.
4. If one is willing to take very high risks of running out of money, say, 25 percent to 30 percent, then withdrawal rates of 5.5 percent to 6 percent can be maintained only at stock allocations of 75 percent to 100 percent. If one is not willing to take such risks, withdrawal rates must be lowered.
5. The statement that a real withdrawal of 4 percent of the initial portfolio is safe is not always correct. Bengen (1994) recommended withdrawal rates of 4 percent with 50/50 stock/bond allocations, and his recommendation is still accurate with the understanding that this combination carries a chance of almost 6 percent of running out of money.
6. The expected amount of the portfolio that remains after 30 years is positively affected by the stock percentage and negatively affected by the withdrawal rate. The balance-remaining contours (Figure 3) demonstrate that the balance remaining may be several times larger than the starting balance (although a zero balance remaining must always be considered) and the amount increases with the stock per-

centage. In terms of balance remaining, higher withdrawal rates can be offset by higher stock percentages while maintaining the same expected balance remaining. The runout contours (Figure 1) serve as a reminder that higher withdrawal rates increase the probability of running out of money before 30 years.

These results provide a wide range of possible choices whereby an individual investor can simultaneously examine the consequences of a "safe" withdrawal rate and "low" risk of running out of money. The runout contour (Figure 1) can address a question such as "Given a 10 percent chance of running out of money, what is the largest amount that can be withdrawn and how should it be allocated between stocks and bonds?" The withdrawal contour (Figure 2) can address a question like, "Given that 4 percent of the starting portfolio balance is withdrawn, how does changing the asset allocation affect the probability of running out of money?" Finally, the balance-remaining contour can help answer a question like this: "If I choose the prudent course of a 4 percent withdrawal and 50 percent stocks, how much might I expect to have left in my portfolio in 30 years?"

Future research might address other scenarios involving more than two asset categories, quarterly or monthly withdrawals, different rebalancing schedules, scenarios that involve non-constant withdrawal rates, or withdrawal schemes where the asset allocation changes during the 30-year period.

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Endnotes

i. The bootstrap allows repeated sampling from a relatively small population but from which statistically valid conclusions may be drawn. One of the 80 years is randomly selected each time; rates of return that *actually* occurred are selected and the differences (spreads) between stock returns and bond returns in that year are historically correct. The number of possible sequences (orderings) of the rates of return is extremely large and the sequence will influence the success or failure of the withdrawal process. Since sampling is with replacement, there are 80 ways to select the first year, 80 ways to select the second year, and so on. All told, there are $80^{30} \approx 1.24*10^{57}$ different 30-year sequences. The bootstrap looks at a mere 10,000 sequences for each of the possible conditions.

2. The average balance remaining is calculated over all 10,000 portfolios for each condition; zero balances are included in the computation. The distribution of balance remaining is right-skewed with a small number of very large portfolios. This oddity makes the mean balance quite large. Since the distribution is right-skewed, less than half the portfolios will experience balance remaining amounts as large as the average amount.

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