Dynamic Allocation Strategies for Distribution Portfolios: Determining the Optimal Distribution Glide Path

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Executive Summary

- The purpose of this paper is to determine the optimal allocation strategy (referred to as the distribution glide path) for a portfolio subject to withdrawals. But unlike most previous research, which uses static allocations, the paper includes a dynamic allocation methodology. It also introduces a methodology to incorporate risk into the decision process.
- Using historical data from four asset categories from 1927 to 2006, 43 different distribution glide paths were considered for 21 different time periods and 61 different real withdrawal rates.
- Despite the expected benefits of more sophisticated dynamic distribution allocation strategies, static equity allocations proved to be remarkably efficient.
- The most optimal glide path from a pure probability-of-success perspective was the 100/0 (100 percent equity and 0 percent fixed income/cash) static allocation portfolio. But due to the underlying variability of a 100/0 portfolio, it is unlikely that this allocation will be appropriate for most retirees.
- The absolute differences in the probability of failure among glide paths for shorter distribution periods and lower real withdrawal rates (less aggressive scenarios) were minor. The absolute differences for longer distribution periods and higher real withdrawal rates (more aggressive scenarios) were considerable.
- The paper introduces a risk-adjusted measure called the Success to Variability ratio in order to incorporate portfolio variability (standard deviation) into the optimal glide path decision process.
- When considering a variety of distribution periods and real withdrawal rates, as well as the probability of failure and the Success to Variability ratio, a balanced static allocation, such as 60 percent equity and 40 percent fixed income/cash, is likely one of the most efficient portfolio allocations for retirees.

Previous research on sustainable real withdrawal rates has focused primarily on appropriate distribution rates given a number of fixed conditions. The purpose of this paper is to provide guidance on sustainable withdrawal rates, as well as to determine the optimal allocation strategy (referred to as the distribution glide path) for a portfolio subject to withdrawals. Unlike previous research, however, this paper also introduces a methodology to incorporate risk (defined as standard deviation) into the optimal portfolio decision process. The most common metric used to gauge the effectiveness of a distribution portfolio is whether it survives the distribution period—its probability of success. While focusing on the probability of success is an approach that certainly has merit, and it is one taken by the author, it ignores the risk of the portfolio necessary to generate the success probability. By combining the underlying risk of the portfolio and its probability of success into one metric (known as the Success to Variability ratio) it becomes possible to compare the overall effectiveness of distribution portfolios by taking both variables into account.

Literature Review

William Bengen is widely regarded as the first person to address the issue of sustainable real withdrawal rates from a financial planning perspective. In his article "Determining Withdrawal Rates Using Historical Data," he found that a "first-year withdrawal rate of 4 percent, followed by inflation-adjusted withdrawals in subsequent

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years, should be safe. In no past case has it caused a portfolio to be exhausted before 33 years." He goes on to analyze the probability of five different equity portfolios (0 percent, 25 percent, 50 percent, 75 percent, and 100 percent) sustaining various withdrawal rates (1 percent to 8 percent). He concludes that the best starting allocation for retirees is an equity allocation between 50 percent and 75 percent, based on historical returns (Bengen 1994).

Additional research by Tezel (2004); Cooley, Hubbard, and Walz (1998); Cassaday (2006); and Guyton and Klinger (2006) confirm the importance of 50 percent-plus equity allocations for distribution portfolios. An exception is Terry (2003), who concludes that higher fixed-income allocations are more optimal than higher equity allocations for a given level of risk despite the fact equity has historically provided a higher expected return.

The distribution periods of previous research have varied, typically ranging between 20 and 40 years, while the sustainable real withdrawal rate generally has been determined to be in the 4–5 percent range. The highest potential recommended real withdrawal rate the author is aware of, without incorporating any type of decision rules, is the 7 percent real withdrawal rate proposed by Cassaday (2006). But Cassaday's "DIESEL" portfolio is clearly optimized in-sample (that is, during the test distribution period). The future out-of-sample ability for such an allocation, or really any allocation, to provide a 7 percent real withdrawal rate is questionable. Cassaday's findings will be addressed in greater depth later in this paper.

Tezel tested a variety of portfolios over three different periods (10, 20, and 30 years) and found that optimal allocations should include domestic large and small equities, as well as government bonds and Treasury bills. Bengen (1994) and Cassaday have also noted the importance of domestic small equities, while the long-term benefits of small-cap equities (as an asset class) have been well documented by French and Fama (1992, 1993, 1995, and 1996). The importance of international equities for distribution portfolios has differed among studies. Cooley, Hubbard, and Walz (2003) find moderate benefit from including international equities, while Kizer (2005) notes a greater benefit.

In an effort to increase the probability of achieving a particular withdrawal rate, a variety of decision rules and more advanced withdrawal strategies have been introduced by Pye (2000), Bengen (2001), Guyton (2004), Guyton and Klinger (2006), and Robinson (2007). Decision rules are relevant from a common-sense perspective: when faced with the possibility of financial ruin, it is likely a retiree will decrease consumption to ensure continued survival of savings. Yet while it is certainly advantageous to create decision rules, since markets and clients (as well as their advisors) can at times be equally irrational, the ability to consistently follow such decision rules over 30 years or more is questionable. Also, dynamic and sophisticated decision rules are not viable strategies for the generally unsophisticated investing public.

Life Expectancy

Before exploring sustainable real withdrawal rates it is important to establish a method to determine the length of the retirement distribution period. Clients not familiar with the nature of life expectancy may question the need to plan for a distribution period of 30 years or more when the current life expectancy for a newborn male is 74.[1] But when life expectancy is viewed correctly, from a probability perspective, it becomes a more dynamic consideration.

Life expectancy is defined as the average number of years of life remaining for a person at a particular age, though many people don't realize that there's a 50 percent chance for the average person to live beyond their average life expectancy. Life expectancies have increased dramatically in developed nations for a variety of reasons, such as improvements in sanitation and nutrition, as well as advances in medical technology.[2] Furthermore, life expectancy is a moving target because it increases as you age. For example, while the life expectancy of a newborn male is 74, the life expectancy for a 74-year-old male is 84—not zero. Consequently, a better way for planners and their clients to determine the length of the distribution period is to determine the acceptable probability of outliving it. Table 1 displays the probability of living to a target age under four different scenarios.

TABLE 1 HERE

Once the acceptable probability of living beyond the distribution period has been decided, it is possible to determine the length of the distribution period. For example, if a male age 65 wanted no more than a 5 percent probability of outliving the projected distribution period, the appropriate distribution period would be until age 95. But if the same individual were willing to accept a 20 percent probability of outliving the distribution period, the appropriate distribution would be only to age 89. The calculation becomes more complex when considering the joint probabilities of a married couple.

For married couples, the probability of either (or both) spouse(s) living beyond the distribution period must be considered. This is a slightly more complex calculation than determining the probability of just one individual living to a certain age. For example, the probability of a male age 60 living to age 95 is only 3.90 percent; however, the probability of at least one spouse of a married couple (both age 60) living to age 95 is 13.18 percent. If this same couple wanted to take only a 5 percent risk that neither spouse would outlive the distribution period, the appropriate distribution period would be until age 98. If only the life expectancy of the wife is considered for a couple both 65 years old (since females have longer life expectancies than males), the projected distribution period would only last until age 84. But there is a 71.38

Table 1: Life Expectancy Probabilities for Various Current and Target Ages

Male Life Expectancy: Probability of Living to Target Age

Target Age	Current Age 50	55	60	65	70	75
60	92%	95%	n/a	n/a	n/a	n/a
65	86%	88%	93%	n/a	n/a	n/a
70	76%	79%	83%	89%	n/a	n/a
75	63%	66%	69%	74%	83%	n/a
80	47%	49%	51%	55%	62%	75%
85	30%	31%	32%	35%	39%	47%
90	13%	14%	14%	16%	18%	21%
95	4%	4%	4%	4%	5%	6%
100	0%	1%	1%	1%	1%	1%

Male and Female (Same Age): Probability of *Both* Spouses Living to Target Age

Target Age	Current Age 50	55	60	65	70	75
60	88%	93%	n/a	n/a	n/a	n/a
65	78%	82%	88%	n/a	n/a	n/a
70	64%	68%	73%	83%	n/a	n/a
75	47%	50%	54%	61%	74%	n/a
80	29%	31%	33%	38%	46%	62%
85	13%	14%	15%	17%	21%	28%
90	3%	4%	4%	4%	5%	7%
95	0%	0%	0%	0%	1%	1%
100	0%	0%	0%	0%	0%	0%

Female Life Expectancy: Probability of Living to Target Age

Target Age	Current Age 50	55	60	65	70	75
60	95%	97%	n/a	n/a	n/a	n/a
65	91%	93%	95%	n/a	n/a	n/a
70	84%	86%	89%	93%	n/a	n/a
75	75%	76%	79%	82%	89%	n/a
80	62%	63%	65%	68%	73%	83%
85	45%	45%	47%	49%	53%	60%
90	25%	26%	26%	28%	30%	33%
95	9%	9%	10%	10%	11%	12%
100	2%	2%	2%	2%	2%	2%

Male & Female (Same Age): Probability of *At Least One* Spouse Living to Target Age

Target Age	Current Age 50	55	60	65	70	75
60	100%	100%	n/a	n/a	n/a	n/a
65	99%	99%	100%	n/a	n/a	n/a
70	96%	97%	98%	99%	n/a	n/a
75	91%	92%	93%	95%	98%	n/a
80	80%	81%	83%	86%	90%	96%
85	61%	62%	64%	67%	71%	78%
90	35%	36%	37%	39%	42%	47%
95	12%	13%	13%	14%	15%	17%
100	2%	2%	2%	3%	3%	3%

Source: Social Security Administration Web site: http://www.ssa.gov/OACT/STATS/table4c6.html., note #1.

percent probability that at least one of two spouses will live past the age of 84.

Stout and Mitchell (2006) introduce the concept of life expectancy when determining an appropriate withdrawal rate. The authors found that while a fixed 4.5 percent withdrawal rate has a 7.16 percent probability of ruin before 30 years, it has a 13.44 percent probability of ruin before life expectancy for someone 60 years old. While it is possible to select the length of the distribution period without considering life expectancy (such as 30 years), considering life expectancy allows a financial planning professional to incorporate probability into the distribution period decision.

Distribution Rates and Probability

The second consideration when determining a sustainable withdrawal rate also relates to probability: determining the acceptable probability of a portfolio failing. Figure 1 includes the maximum withdrawal rates for 11 different equity allocations (decreasing from 100 percent to 0 percent in 10 percent increments) for 20 different distribution periods (ranging from 20 to 40 years in 1-year increments) based on a 5 percent probability of failure. Information on the calculation methodology and the assumptions is discussed at length in the Analysis section of this paper.

As noted earlier, a male age 65 who wants only a 5 percent probability of outliving the distribution period would select a target life expectancy of 95 (or a distribution period of 30 years, assuming the beginning and ending years are not inclusive). Also assume this same male wanted a portfolio of 60 percent stocks and 40 percent bonds. Based on the information in Figure 1, his real maximum withdrawal rate would be 4.1 percent. The withdrawal rate is defined in "real" terms such that the effects of inflation have been removed.

Something interesting to note about Figure 1 is the "humped" nature of the distribution. While one might expect the equity allocations at either extreme (100/0 or 0/100) to have the highest real withdrawal rates, this was not the case (based on a 5 percent probability of failure). In fact, the balanced allocations (such as 50/50 and 40/60) had the highest potential real withdrawal rates for almost every distribution scenario. Therefore, those retirees only willing to accept a 5 percent probability of failure are likely best served with a balanced allocation, regardless of

Figure 1: Maximum Real Withdrawal Rates for a Variety of Equity Allocations with a 5 Percent Probability of Failure

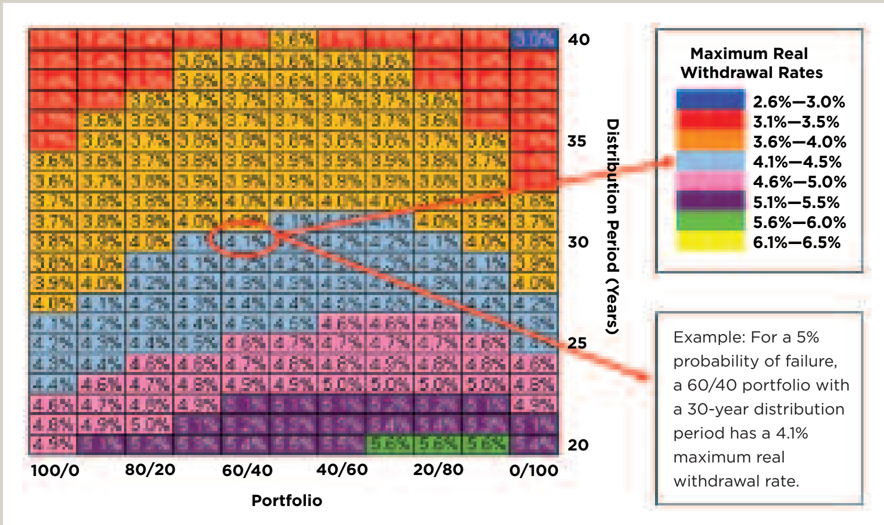
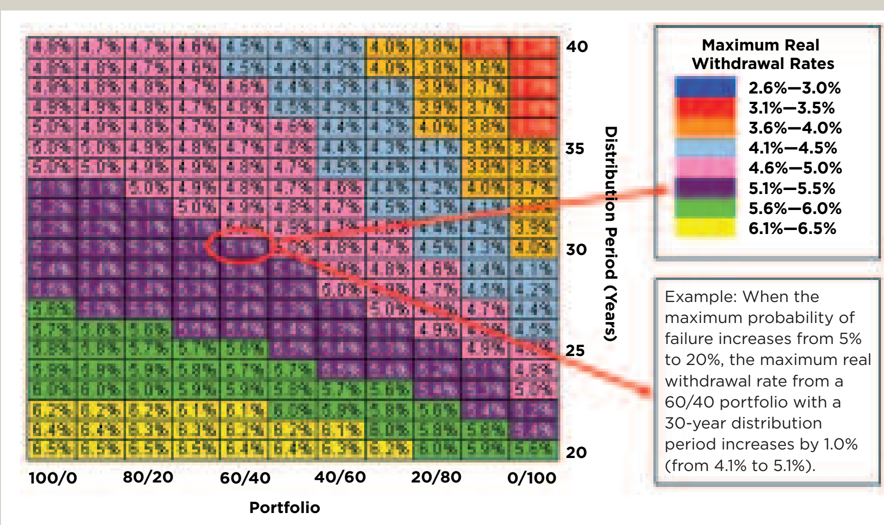


Figure 2: Maximum Real Withdrawal Rates for a Variety of Equity Allocations with a 20 Percent Probability of Failure



their expected distribution period.

What if, however, a retiree is willing to accept a 20 percent probability of distribution failure? Figure 2 includes the same general information as Figure 1, except the probability of failure has been increased from 5 percent to 20 percent.

Notice that unlike Figure 1, the highest real withdrawal rates based on a 20 percent probability of failure are for those portfolios with higher equity allocations. While the more balanced (such as 50/50) portfolios had the highest real withdrawal rates based on a 5 percent probability of failure, the more aggressive portfolios resulted in higher real withdrawal rates when considering a greater likelihood of failure. The increase in the real withdrawal rates for all scenarios from the 0/100 portfolios to the 100/0 portfolios averaged 1.3 percent. The highest real withdrawal rate (6.5 percent) was for the 100/0 portfolio with a 20-year distribution period, while the lowest real withdrawal rate (3.2 percent) was for the 0/100 portfolio with a 40-year distribution period. The differences in Figures 1 and 2 underscore the importance of determining an acceptable probability of failure for a retiree.

Analysis

The term *glide path* is commonly used to describe the decreasing equity allocation for target-date mutual fund investments. As the target-date investment approaches its retirement date, the overall equity allocation tends to decrease, at an increasing rate (taking the general shape of a concave hyperbola). For this analysis, "glide path" is used to describe the equity allocation throughout the entire distribution period. While past research on sustainable withdrawal rates has tended to focus on more distinct time periods (say 20, 30, or 40 years), this analysis will increase the scope of the distribution period by analyzing distribution periods from 20 to 40 years (in one-year increments) and real withdrawal rates from 3 percent to 8 percent (in .1 percent increments), for a total of 1,071 scenarios.

The rates of return and standard deviations for the asset categories considered for any analysis on sustainable withdrawals will have a dramatic impact on the results. In the attempt to minimize the impact of time-period selection bias, the longest period of returns available to the author was used for the analysis (1927–2006, or 80 years of data). Monthly data from 1927 until 2006 (960 months) was obtained on four asset categories:[3]

1. Cash
2. Intermediate-term bond
3. Domestic large-blend equity
4. International equity

The monthly returns were adjusted into real terms by subtracting the monthly inflation rate, which was defined as the increase in the Consumer Price Index for all Urban Consumers (CPI-U).[4] Real

Table 2: Annualized Real Returns and Annual Standard Deviations for Selected Asset Categories

	Geometric Real Returns	Arithmetic Real Return	Standard Deviation
Cash	0.72%	0.81%	4.10%
Intermediate-term Bond	2.34%	2.52%	6.14%
Domestic Large Blend Equity	7.36%	9.48%	20.99%
International Equity	5.16%	7.25%	21.59%

Data Source: See Endnotes 3.

returns were considered because people typically seek to maintain a constant level of purchasing power as prices increase with inflation. While there are different types of inflation (such as health care inflation), the CPI-U was used because it is the most common definition of inflation. The average annual geometric inflation rate for the period tested (1927–2006) was 3.51 percent. The annual real returns and annual standard deviations for the four asset categories considered in the analysis are included in Table 2.

The 9.44 percent average arithmetic real return for large-blend equity over the test period was similar to the 9.17 percent return calculated by Stout and Mitchell (2006) using Ibbotson Associates data over a similar test period (1926–2004). While the historical performance represents the returns an investor could have achieved (before fees) had he or she been able to invest in these asset categories, these were not investable indexes for the entire historical period (for example, it would have been impossible to buy the International Equity proxy).

Some readers may question the exclusion of domestic small-cap equities from consideration in the portfolios despite previous research noting their benefits for distribution portfolios (refer to the Literature Review section, earlier in this article). The reason for their exclusion was due to their high return over the test period. Small-cap equities (defined using the return information available on Kenneth French's Web site) had a 10.46 percent geometric annualized real return (13.89 percent nominal) and a 14.10 percent arithmetic annualized real return over the test period (17.56 percent nominal). Such high returns seem aggressive on a forward-looking basis and would result in an upward bias in the available sustainable withdrawal rate in a portfolio if these returns were not realized in the future.

For the portfolio allocations, the ratio between cash and intermediate-term bond and between domestic large blend equity and international equity was held constant regardless of the overall cash/fixed income and equity allocation. The allocation between cash and intermediate-term bond was split 50 percent each, while the allocation between domestic large-blend equity and international equity was split 66.67 percent and 33.33 percent, respectively. For example, a 60/40 portfolio would have a 20 percent cash allocation, a 20 percent intermediate-term bond allocation, a 40 percent domestic large-blend equity allocation, and a 20 percent international equity allocation.

While the test allocation may be viewed as overly simplistic, it is important to note that portfolio theory is constantly changing. Allocations recommended to clients today are considerably different than those recommended 20 years ago and will likely again be different from those implemented 20 years from now. Using overly precise allocations that performed well historically (Cassaday's DIESEL allocation, for example) can lead to overly optimistic withdrawal assumptions. If the original research by Brinson, Hood, and Beebower (1986), and more recently by Tokat, Wicas, and Kinniry (2006), has taught us anything, it is that the overall equity allocation decision should be the primary focus during the portfolio construction process, not the more precise allocations to different asset categories.

The actual returns used for testing purposes were created through a process known as bootstrapping. Bootstrapping is a type of simulation analysis where the in-sample test period returns are randomly recombined to create sample annual returns. For the analysis, the 960 monthly returns were randomly recombined to create hypothetical real annual rates of return for the analysis. For example, the monthly real returns for each of the four categories for the same month (for example, June 1961) would be recombined with monthly real returns from 11 other months (for example, March 1930, January 1995, May 1979, and so on) to create each hypothetical annual real return.

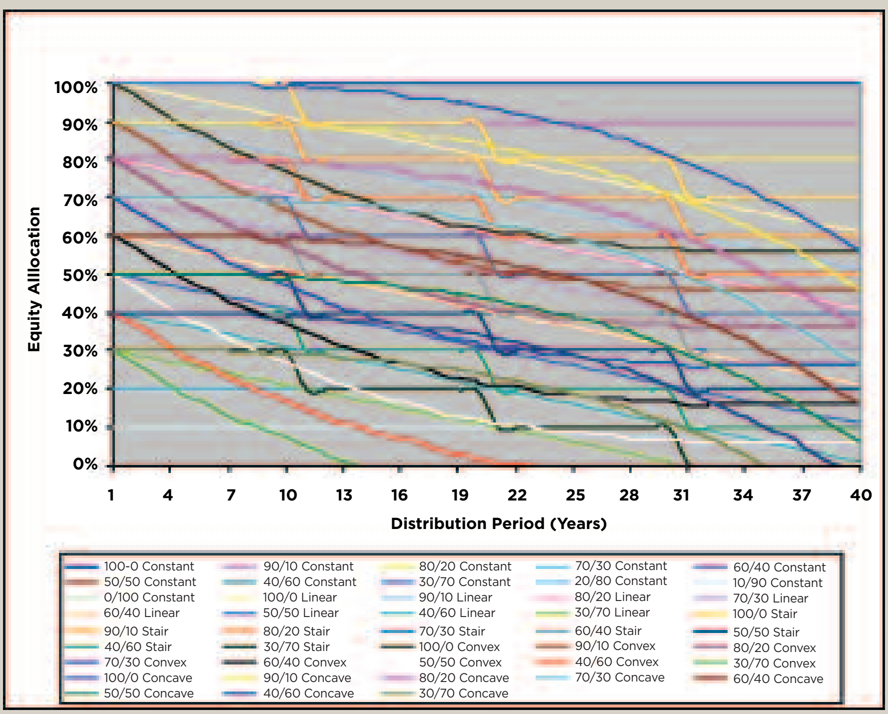
A benefit of using the actual historical monthly returns (through bootstrapping) is that no assumption needed to be made regarding the distribution of returns (such as lognormal or leptokurtic). A potential problem with bootstrapping, however, is that it assumes that the cross correlations among asset categories are maintained for each recombined bootstrapped sub-period. But since the recombination period was small (monthly) and the recombination sample was large (960 months spanning 80 years), this was not considered to be a significant issue.

The annual distribution was assumed to have been taken from the portfolio once a year, at the beginning of each year. Each test scenario (for example, 60/40 Constant portfolio, 30-year distribution period, and 4 percent real withdrawal rate) was subjected to a 10,000 run Monte Carlo simulation. Note that the Monte Carlo simulator (that is, bootstrap simulator) used for the analysis was built in Microsoft Excel since the author was not aware of any existing programs that could incorporate dynamic asset allocation changes (on an annual basis) during the distribution period. Also, it was important that the test used geometric returns instead of arithmetic returns, since arithmetic returns with higher standard deviations are biased downward on a

Table 3: Beginning Allocations (Equity/Fixed) and Names of the 43 Different Distribution Glide Paths Tested

Constant	Linear	Stair	Convex	Concave
0/100 Constant	30/70 Linear	30/70 Stair	30/70 Convex	30/70 Concave
10/90 Constant	40/60 Linear	40/60 Stair	40/60 Convex	40/60 Concave
20/80 Constant	50/50 Linear	50/50 Stair	50/50 Convex	50/50 Concave
30/70 Constant	60/40 Linear	60/40 Stair	60/40 Convex	60/40 Concave
40/60 Constant	70/30 Linear	70/30 Stair	70/30 Convex	70/30 Concave
50/50 Constant	80/20 Linear	80/20 Stair	80/20 Convex	80/20 Concave
60/40 Constant	90/10 Linear	90/10 Stair	90/10 Convex	90/10 Concave
70/30 Constant	100/0 Linear	100/0 Stair	100/0 Convex	100/0 Concave
80/20 Constant				
90/10 Constant				
100/0 Constant				

Figure 3: Distribution Glide Paths Tested



geometric-return basis.

A portfolio was considered successful if it did not run out of money during the distribution period. The amount of the ending balance was not considered. The portfolios were assumed to be held in tax-deferred accounts and any tax implications of the withdrawals were ignored. Based on the bootstrapping methodology, it is implicitly assumed that the portfolios were rebalanced back to their target allocations monthly. Any potential costs associated with the rebalancing were ignored. Tax considerations were also ignored for the analysis.

Test Distribution Glide Paths

Five primary types of distribution glide path strategies were tested. Table 3 includes each of the 43 glide paths tested.

1. Constant: static allocation for the entire period.
2. Linear: the equity allocation decreases by 1 percent a year throughout the distribution period.
3. Stair: the equity allocation decreases by 10 percent each ten years throughout the distribution period.
4. Concave: the equity allocation decreases at an increasing rate and resembles a concave hyperbola throughout the distribution period.[5]
5. Convex: the equity allocation decreases at a decreasing rate and resembles a convex hyperbola throughout the distribution period.[6]

Any type of equity reduction for the distribution glide paths was assumed to begin during the second distribution year. For example, a 70/30 Linear strategy would have a 70 percent allocation during the first year, a 69 percent allocation in the second year, a 68 percent allocation in the third year, and so on. Figure 3 includes a chart that contains the equity allocations for each of the 43 different distribution glide paths considered for the analysis over the entire 40-year distribution period.

The distribution glide paths tested represent a number of potential equity reduction strategies. To give the reader a scope of the analysis conducted for this paper, each of the 43 different test glide paths was tested for distribution periods between 20 and 40 years (in one-year increments, inclusive) for withdrawal rates between 3 percent and 8 percent (in .1 percent increments, inclusive) at 10,000 runs per scenario. This means that each glide path had a total of over 10 million different test runs, and the overall analysis conducted over 450 million different test runs.

Distribution Glide Paths: General Results

Table 4 contains the results for 9 of the 1,071 scenarios tested. While the paper will later address a more thorough analysis covering each of the tested scenarios, Table 4 provides the reader with a general insight

Table 4: Probabilities of Failure for a Variety of Scenarios for each of the 43 Distribution Glide Paths

	Scenario								
Distribution Rate	**4.00%**	**4.00%**	**4.00%**	**5.00%**	**5.00%**	**5.00%**	**6.00%**	**6.00%**	**6.00%**
Distrib Period (Years)	**20**	**30**	**40**	**20**	**30**	**40**	**20**	**30**	**40**
Glideppath	**Probability of Failure**								
0/100 Constant	0.00%	14.81%	98.25%	0.08%	99.67%	100.00%	63.99%	100.00%	100.00%
10/90 Constant	0.00%	3.01%	65.53%	0.07%	84.03%	99.94%	26.35%	99.99%	100.00%
20/80 Constant	0.00%	2.10%	32.77%	0.10%	52.53%	93.44%	16.76%	96.60%	99.93%
30/70 Constant	0.00%	2.06%	18.07%	0.51%	32.55%	70.74%	12.51%	81.27%	96.41%
40/60 Constant	0.00%	2.66%	13.73%	1.02%	24.44%	53.35%	11.79%	65.16%	86.42%
50/50 Constant	0.09%	3.29%	10.97%	1.75%	19.28%	37.96%	11.16%	49.84%	70.06%
60/40 Constant	0.17%	3.58%	10.13%	2.23%	17.52%	32.61%	11.38%	42.49%	60.35%
70/30 Constant	0.33%	4.21%	9.77%	3.16%	16.34%	27.10%	11.97%	36.16%	50.14%
80/20 Constant	0.77%	4.64%	9.42%	3.80%	15.66%	25.06%	12.20%	33.10%	44.21%
90/10 Constant	1.00%	5.39%	9.83%	4.52%	15.48%	23.30%	12.62%	30.58%	40.58%
100/0 Constant	1.56%	6.10%	10.11%	5.14%	15.58%	22.26%	13.32%	28.80%	37.55%
30/70 Linear	0.00%	3.64%	38.57%	0.50%	49.28%	92.76%	16.16%	93.75%	99.69%
40/60 Linear	0.00%	3.31%	23.10%	0.77%	33.93%	72.58%	13.84%	77.20%	95.75%
50/50 Linear	0.01%	3.59%	16.16%	1.50%	25.31%	53.98%	12.45%	61.54%	84.58%
60/40 Linear	0.13%	4.26%	13.62%	2.41%	21.29%	41.65%	12.72%	50.02%	70.94%
70/30 Linear	0.45%	4.56%	11.85%	3.04%	18.39%	34.00%	12.39%	41.92%	60.03%
80/20 Linear	0.52%	4.86%	10.61%	3.73%	17.04%	28.81%	12.42%	36.29%	51.05%
90/10 Linear	0.92%	6.71%	13.78%	4.31%	17.77%	28.22%	12.94%	34.05%	45.04%
100/0 Linear	1.36%	5.81%	10.84%	4.99%	15.98%	24.09%	13.42%	30.43%	40.74%
30/70 Stair	0.00%	3.07%	29.14%	0.41%	40.83%	85.09%	14.00%	87.91%	98.99%
40/60 Stair	0.00%	3.09%	18.78%	0.92%	28.54%	64.24%	12.21%	70.71%	92.45%
50/50 Stair	0.06%	3.55%	14.16%	1.63%	22.06%	47.13%	12.00%	55.03%	78.16%
60/40 Stair	0.11%	3.82%	12.02%	2.06%	18.90%	37.52%	11.61%	46.10%	65.86%
70/30 Stair	0.36%	4.51%	11.29%	3.10%	17.68%	31.30%	12.62%	38.75%	54.69%
80/20 Stair	0.67%	4.97%	10.34%	3.68%	16.19%	27.37%	12.36%	34.58%	47.73%
90/10 Stair	1.16%	5.90%	10.82%	4.75%	16.04%	25.19%	13.23%	32.34%	43.64%
100/0 Stair	1.51%	6.04%	10.45%	5.16%	15.94%	23.23%	13.31%	29.69%	39.12%
30/70 Convex	0.00%	6.86%	67.72%	0.27%	74.96%	99.44%	25.13%	99.33%	100.00%
40/60 Convex	0.00%	5.50%	45.66%	0.87%	53.30%	92.78%	18.18%	92.56%	99.55%
50/50 Convex	0.01%	4.73%	28.93%	1.19%	36.46%	74.87%	15.39%	77.10%	95.50%
60/40 Convex	0.04%	4.71%	19.40%	2.08%	27.23%	57.15%	13.86%	62.40%	85.48%
70/30 Convex	0.13%	4.87%	15.38%	2.93%	22.55%	43.35%	14.04%	49.96%	72.12%
80/20 Convex	0.33%	4.99%	12.57%	3.53%	19.32%	35.29%	12.64%	42.11%	60.59%
90/10 Convex	0.71%	5.50%	11.96%	4.03%	17.52%	29.62%	13.12%	36.84%	52.04%
100/0 Convex	1.05%	5.99%	11.43%	4.92%	17.15%	26.77%	13.56%	33.48%	46.07%
30/70 Concave	0.00%	2.45%	24.14%	0.47%	36.22%	78.64%	13.46%	84.05%	97.93%
40/60 Concave	0.01%	2.99%	16.69%	0.95%	26.23%	59.31%	12.21%	67.73%	89.26%
50/50 Concave	0.08%	3.25%	12.39%	1.63%	19.68%	42.50%	11.14%	51.09%	73.69%
60/40 Concave	0.04%	4.71%	19.40%	2.08%	27.23%	57.15%	13.86%	62.40%	85.48%
70/30 Concave	0.36%	4.54%	10.55%	3.12%	16.54%	29.19%	11.75%	37.25%	52.02%
80/20 Concave	0.58%	4.82%	10.20%	3.74%	15.93%	26.19%	12.60%	33.93%	46.50%
90/10 Concave	1.03%	5.55%	10.30%	4.50%	15.56%	24.13%	12.70%	30.64%	41.22%
100/0 Concave	1.55%	6.15%	10.37%	5.13%	15.70%	22.69%	13.35%	29.13%	38.23%
Minimum	0.00%	2.06%	9.42%	0.07%	15.48%	22.26%	11.14%	28.80%	37.55%
Maximum	1.56%	14.81%	98.25%	5.16%	99.67%	100.00%	63.99%	100.00%	100.00%
Range	1.56%	12.75%	88.83%	5.09%	84.19%	77.74%	52.85%	71.20%	62.45%

Best glide path for scenario (green)   Worst glde path for scenario (blue)

into the overall efficiency of the 43 different distribution glide paths for a select few scenarios. The best distribution glide path for each scenario is highlighted in green, while the worst glide path for each scenario is highlighted in red. Note, if multiple glide paths had the same probability of failure and were either the best or worst for the scenario (for example, a 4 percent real withdrawal rate and a 20-year distribution period), they would both be highlighted.

A variety of general conclusions can be reached from Table 4.

The best distribution glide path now might be the worst distribution glide path later. The distribution period and the real withdrawal rate can be extremely important when determining the optimal glide path. For example, the 0/100 Constant portfolio was one of the best glide paths for the 4 percent real withdrawal rate and 20-year distribution period but was the worst glide path for the 4 percent real withdrawal rate and the 40-year distribution period. Therefore, studies based on different distribution periods (say 20 years versus 40 years) are likely have varying conclusions if multiple periods are not considered.

Die soon enough and the allocation doesn't matter. The absolute differences in the probability of failure for lower real withdrawal rates and for shorter distribution periods (less aggressive scenarios) was minor compared with the differences in the probability of failure for higher real withdrawal rates and longer distribution periods (more aggressive scenarios). For example, the range of failure probabilities for the 20-year distribution period with a 4 percent real withdrawal rate was only 1.56 percent (between .00 percent and 1.56 percent), while the range of failure probabilities for the 40-year distribution period with a 6 percent real withdrawal rate was 62.45 percent (between 37.55 percent and 100 percent). This represents an extreme difference in probability of success and underscores the importance of using higher equity allocations, at least initially, during the distribution period.

Static allocations did very well. The Constant (static) distribution glide paths were remarkably efficient despite their simplicity. While one would think that a more dynamic asset allocation strategy (such as the Concave distribution glide path) might result in a lower probability of failure (higher probability of success), the Constant portfolios had the lowest probabilities of failure for all but one of the above test scenarios (apart from the 20-year, 4 percent scenario, which had multiple "best" distribution glide paths).

While the probabilities of failure tended to be slightly lower than previous research on an absolute basis, the overall results of the analysis were similar. For example:

1. Stout and Mitchell (2006) found that a 65/35 Constant allocation, assuming a 4.5 percent real withdrawal rate and a 30-year distribution period, has a 13.44 percent probability of failure over 30 years. This author found a 65/35 Constant allocation (not included above) has a 10.04 percent probability of failure (an absolute difference of 3.40 percent).
2. Ameriks and Warshawsky (2001) found that a 60/40 Constant allocation, assuming a 4.5 percent real withdrawal rate and a 30-year distribution period, has a 12.60 percent probability of failure in 30 years. Based on the same criteria, this author determined the probability to be 8.72 percent, for an absolute difference of 3.88 percent.
3. Bengen (2001) found that a 63/37 Constant allocation, assuming a 5 percent real withdrawal rate and a 30-year distribution period, has a 19 percent probability of failure in 30 years. This author found a 60/40 Constant allocation has a 17.52 percent probability of failure (an absolute difference of 1.48 percent).

The greatest disparity among the results of this analysis and previous published research was by Cassaday. He contends that his DIESEL portfolio is able to sustain a 7 percent withdrawal rate over a 33.5-year distribution period with a 9.1 percent probability of failure. The most similar distribution glide path tested for this analysis, the 80/20 Constant, had a 57.92 percent probability of failure over a 34-year distribution period. This represents a probability of failure that was *six times greater* than the probability of failure determined by Cassaday.

Two key problems with Cassaday's research were the inflation assumption and the clearly in-sample optimized portfolio. First, Cassaday assumed a 3 percent inflation rate for the test period (January 1, 1972, through July 31, 2005), despite the fact that the actual geometric annualized inflation rate during the test period (as measured by the increase in the CPI-U) was 4.75 percent. Second, while Cassaday's DIESEL allocation clearly performed well historically (that is, in-sample), the out-of-sample (future) benefits of such are still unknown. Determining the best historical allocations is easy—predicting which allocations will do best in the future is an entirely different matter.

Best Distribution Glide Paths

For the general results, the 100/0 Constant was the distribution glide path that had the lowest probability of failure for the nine scenarios considered in the General Results. This relationship was consistent when all 1,071 scenarios were considered. Figure 4 shows the best distribution glide path for each distribution period and withdrawal rate tested. The color for each scenario (or cell) represents the respective optimal glide path for that scenario. For example, the color pink, which represents the 30/70 Constant distribution glide path, covers the 4.0 percent real withdrawal rate for the 30-year distribution period. This means the 30/70 Constant distribution glide path was the glide path with the lowest probability of failure (or highest probability of success) among the 43 glide paths tested for that scenario.

Of the 43 different distribution glide paths considered, only 14 were distinctly better than the other glide paths for any given scenario. If multiple glide paths had the same (lowest) probability of failure for a given scenario, the term "Multiple" is used since no one methodology was optimal. The Multiple category was prevalent primarily for only the least aggressive scenarios (that is, the ones with the lowest withdrawal rates and the shortest distribution periods).

While 100/0 Constant was the most common optimal distribution glide path, the 0/100 Constant distribution glide path was not uniquely the best for any of the 43 distribution glide paths. The lack of congruence among the portfolios for the different test scenarios was also somewhat alarming. For example, there is a "jump" for scenarios where the 30/70 Constant portfolio is the optimal distribution glide path to where the 80/20 Constant is the optimal glide path if the real withdrawal rate is increased by a mere .2 percent or the distribution period is increased by two years. The long-term differences in both return and risk for a 30/70 and an 80/20 portfolio are considerable. This suggests that a more balanced allocation, with a slightly higher equity allocation, such as a 60/40 Constant distribution glide path, would likely be a more practical allocation for retirees compared with either extreme.

Best Distribution Glide Paths Excluding Constant Portfolios

Because an equity reduction strategy is a likely methodology for a number of retirees who will want to reduce their equity exposure (risk, in other words) throughout retirement, a second analysis was conducted to determine the optimal distribution glide path where the Constant glide paths were removed. Figure 5 contains the results of this analysis.

When the Constant portfolios are removed, the 100/0 Concave portfolio proved to be the most optimal distribution glide path and the Concave methodology was clearly the best distribution strategy. The 100/0 Concave glide path was the best glide path for 55.93 percent of all scenarios, while the Concave methodology accounted for 89.47 percent of scenarios that were not considered Multiple. The Concave methodology has an equity reduction methodology that is highest at the beginning and then starts to decrease at a faster and faster rate throughout the distribution period. The shape of the Concave glide path is very similar to the shape of the glide path for most target-date mutual funds.

Introducing the Success to Variability Ratio[7]

From a probability-of-failure perspective, the Constant portfolios proved to be far from inefficient. If one were to select a portfolio based entirely on the probability of success, the 100/0 Constant distribution glide path (the most aggressive glide path tested) would likely be considered the most optimal. This is because although the 100/0 Constant glide path tended to have the lowest probability of success for shorter distribution periods and lower real withdrawal rates, the absolute differences were small for those scenarios, and it proved to be a much more advantageous strategy (on an absolute basis) for longer distribution periods and higher real withdrawal rates.

But selecting an optimal distribution

Figure 4: Maximum Real Withdrawal Rates for a Variety of Equity Allocations with a 5 Percent Probability of Failure

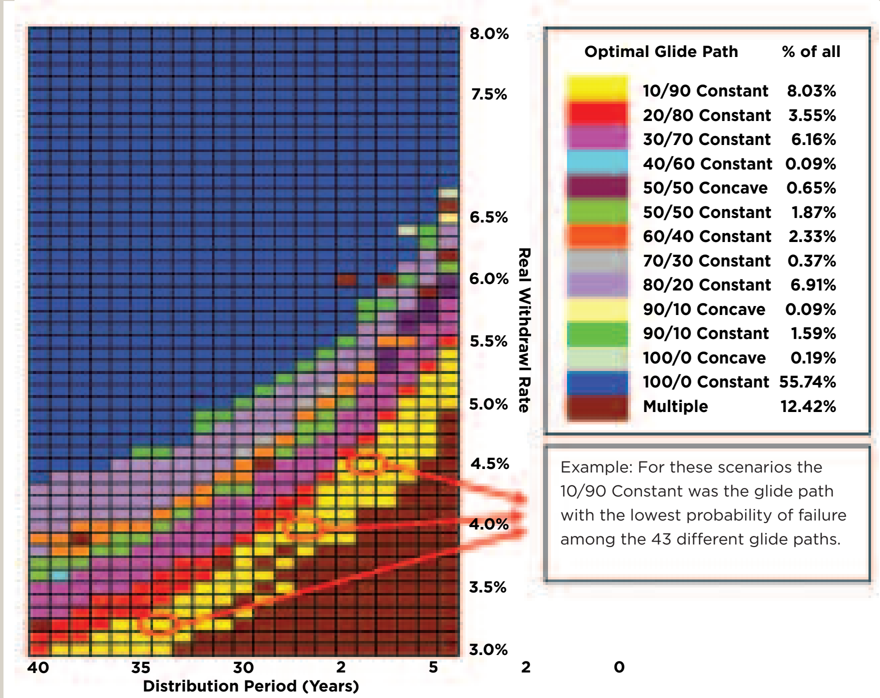


Figure 5: Best Distribution Glide Paths Excluding Constant Portfolios for Each of the 1,071 Test Scenarios Based on the Lowest Probability of Failure

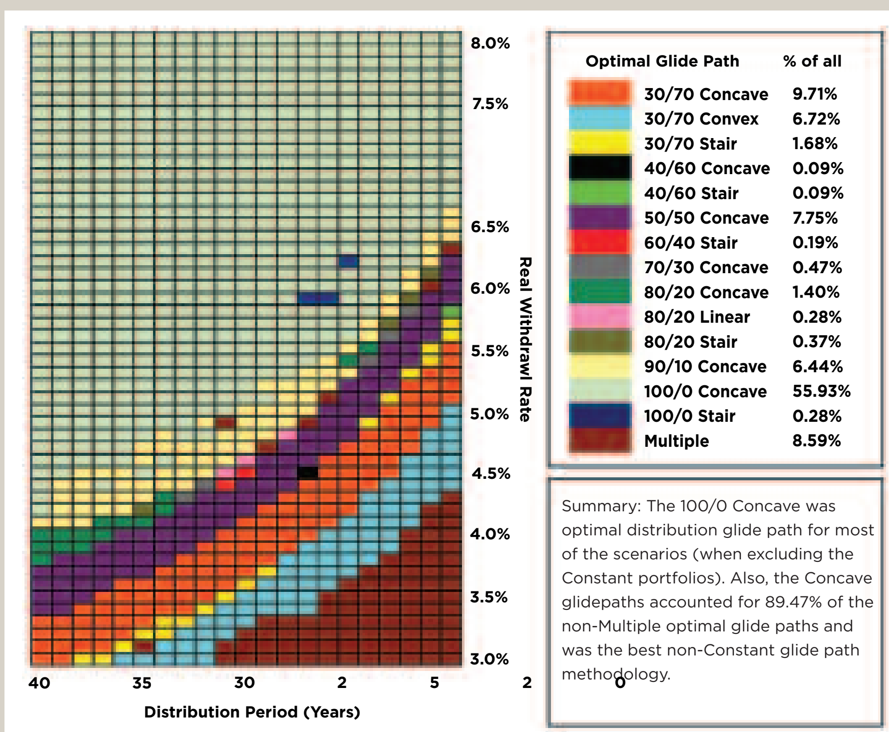


Figure 6: Best Constant Distribution Glide Paths for each of the 1,071 Test Scenarios Based on the Highest Success to Variability Ratio

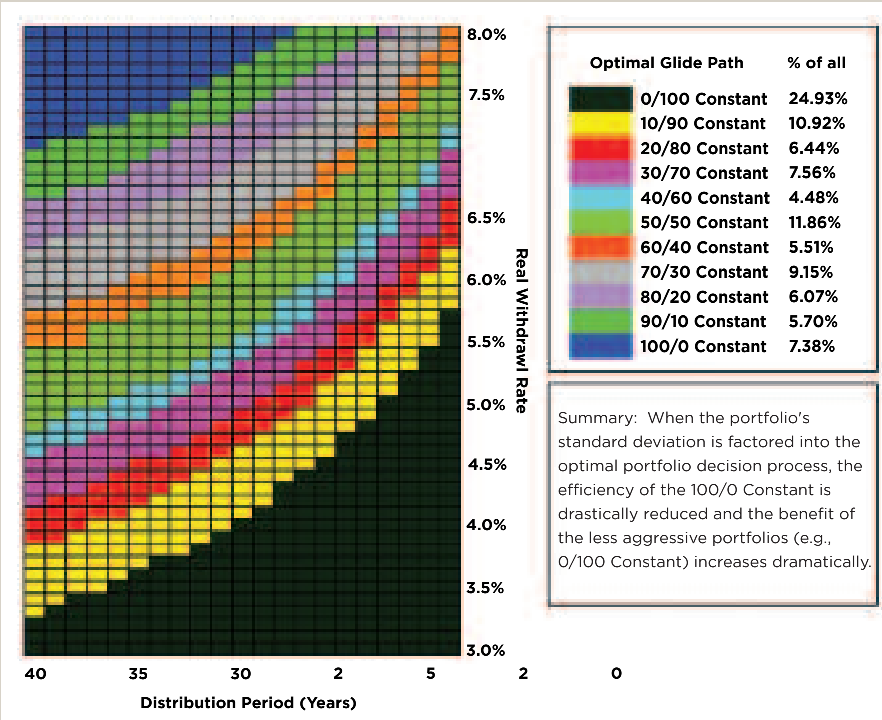


Table 5: Comparison Between the Probability of Success and the Success to Variability Ratio for the Optimal Portfolio Decision

Glide Path	Portfolio Standard Deviation	Pure Probability of Success		Success to Variability Ratio	
		#	%	#	%
0/100 Constant	2.20%	0	0.00%	267	24.93%
10/90 Constant	2.76%	86	8.03%	117	10.92%
20/80 Constant	3.96%	38	3.55%	69	6.44%
30/70 Constant	5.49%	66	6.16%	81	7.56%
40/60 Constant	6.93%	1	0.09%	48	4.48%
50/50 Constant	8.56%	28	2.61%	127	11.86%
60/40 Constant	10.04%	27	2.52%	59	5.51%
70/30 Constant	11.92%	4	0.37%	98	9.15%
80/20 Constant	13.72%	74	6.91%	65	6.07%
90/10 Constant	15.31%	18	1.68%	61	5.70%
100/0 Constant	17.08%	600	56.02%	79	7.38%
Multiple	n/a	129	12.04%	0	0.00%
	Total	**1,071**	**100.00%**	**1,071**	**100.00%**

glide path based entirely on the probability of success ignores the underlying risk associated with the portfolio. The 100/0 Constant distribution glide path has nearly seven times the variability (or risk) of the 0/100 Constant glide path (17.08 percent versus 2.20 percent). While retirees are concerned with having enough money during retirement (that is, a high probability of success), they are also concerned with the variability of their portfolios. High portfolio variability can lead to sleepless nights for clients, which can then result in poor investment decisions (exiting the market after a large loss). Therefore, the underlying variability of a portfolio should be considered when selecting the optimal portfolio for a retiree.

To incorporate variability into the optimal distribution glide path decision, a risk-adjusted method for determining the optimal Constant distribution glide path is introduced. This method, called the Success to Variability ratio, uses standard deviation as the definition of risk because standard deviation is the most common definition of risk among investment professionals.[8] Although financial markets exhibit non-normal characteristics, such as kurtosis and skewness, standard deviation is nevertheless a useful tool to describe the distribution of returns for investments financial markets (DiBartolomeo 1993). Based on logic similar to that of the Sharpe ratio,[9] the numerator is adjusted (using the probability of success instead of the excess return) to create a success-to-variability ratio:

$$\text{Success to Variability Ratio} = \frac{\text{Probability of Success}}{\text{Standard Deviation of the Portfolio}}$$

The Success to Variability ratio allows the user to assign a risk-adjusted score to a distribution portfolio, which considers both the probability of success and the underlying risk of the portfolio. The higher the Success to Variability ratio score, the more optimal the portfolio. As an example, the 100/0 Constant distribution glide path was very efficient on a probability-of-success basis. But when variability is included in the optimal decision process, it becomes much less optimal. Figure 6 includes the optimal Constant distribution glide paths based on the highest Success to Variability ratio for each of the 1,071 scenarios tested. Table 5 compares the overall distribution of the portfolios. Note, Figure 4 provides a good reference for Figure 6 if the reader is interested in comparing the Success to Variability ratio results to the pure probability of success results.

When the Success to Variability ratio is used to define the optimal distribution glide path, the less aggressive portfolios become much more efficient. The 0/100 Constant glide path had the highest Success to Variability ratio for most of the 1,071 scenarios (267 or 24.93 percent). In contrast, the 0/100 Constant glide path was never defined as the uniquely optimal portfolio based entirely on the probability of success. The 100/0 Constant distribution glide path, which had the highest probability of success for 56.02 percent of the scenarios, was only optimal on a Success to Variability ratio basis for 7.38 percent of the scenarios.

The difference between the optimal portfolio definitions for the two methods emphasizes the importance noted earlier of taking a balanced approach to selecting the equity allocation for a client. While the 100/0 Constant distribution portfolio was the portfolio with the highest overall probability of success and the 0/100 Constant distribution portfolio was the portfolio with the highest overall Success to Variability Ratio, neither extreme is likely the best for most clients. Instead, a more balanced approach to investing, such as a 60/40 portfolio, is likely a good starting place for most retirees since it has a good balance of success and risk.

Limitations with the Success to Variability Ratio

While the Success to Variability ratio provides the user with a methodology to introduce risk in the optimal distribution portfolio process, it has three limitations:

1. It would be difficult to use the Success to Variability ratio for a distribution glide path with a non-constant equity allocation. This is because unlike the Constant, or static glide paths, non-constant glide paths (such as Concave) would have non-constant standard deviations throughout the distribution period.
2. While standard deviation is the most common definition of risk for investment purposes, investors do not fear making too much money (upside deviation), which is why other definitions of risk (such as downside risk) may prove to be more useful.
3. The difference between a probability of failure of 5 percent and 10 percent, or the difference between a standard deviation between 6 percent and 9 percent, is not constant across investors. Incorporating more advanced risk-matching strategies (such as Lower Partial Moments) may prove useful in solving this problem.

Despite its limitations, the Success to Variability ratio is a useful tool in determining the optimal distribution portfolio. While future research is likely necessary to further refine the concept, the concept of considering both the probability of success and the underlying risk of a portfolio is an important one.

Our Clients and Our Profession

A key assumption of the analysis was that the distribution glide paths tested were followed for the entire retirement period. Given the demographics of our profession and the often irrational nature of clients, this may be a questionable assumption. Currently, the average approximate age of a CFP practitioner is 49 and only 25 percent of CFP certificants are under the age of 40.[10] If we assume a distribution period of 20-plus years, it is likely that only a handful of financial planning professionals who determine the initial withdrawal strategy will likely be around to determine the strategy's success.

Furthermore, the analysis assumed that a *client* actually sees the distribution glide path strategy through until completion. Adverse market conditions can lead to irrational investing, such as buying high and selling low, which is common among individual investors (one only needs to refer to one of the many DALBAR studies). Such behavior can have a dramatic negative impact on the success of any given distribution strategy. This suggests that equity allocations should not be overly aggressive and that the underlying variability of the portfolio should be considered when determining the optimal distribution portfolio for a client.

Fees and Expenses

Investment fees and expenses such as advisory fees and fund expenses must be taken into consideration when projecting a real withdrawal rate because, like the retiree's distribution income, such fees represent fixed outflows that must be deducted from a portfolio regardless of market conditions. Although this is a topic that was covered at length by Pye (2001), it is important to understand the impact that fees and expenses can have on retirement income when determining an appropriate withdrawal rate for a retiree.

For example, if a retiree needs a 4 percent real withdrawal rate for 30 years, the probability of failure for a 60/40 static portfolio is only 3.58 percent. But if we assume a 1 percent advisory fee and an average expense ratio of .5 percent (for an aggregate cost of 1.5 percent), the real withdrawal rate necessary to meet the client's 4 percent distribution goal would need to be increased by 1.5 percent to 5.5 percent (ignoring any additional tax considerations).[11] The probability of failure for a 5.5 percent real withdrawal rate and a 30-year distribution period (with a 60/40 portfolio) is 28.99 percent, which is more than eight times the probability failure for the 4 percent withdrawal rate. If the client's initial distribution goal was 5 percent, there is a 17.52 percent probability of failure; however, a real withdrawal rate of 6.5 percent (again assuming an increase of 1.5 percent for fees and expenses) has a probability of failure of 56.32 percent. In summary, fees and expenses are very important and must be considered during the sustainable real withdrawal rate for a client portfolio.

Conclusion

Despite their simplicity, Constant (static allocation) distribution glide paths proved to be remarkably efficient distribution strategies, followed by the Concave distribution glide path strategy. But in order to determine the true optimal distribution glide path, a variety of factors must be considered. If the distribution period and real withdrawal rate are both known, it is possible to determine the optimal distribution glide path based entirely on probability of failure. However, focusing entirely on the probability of failure ignores the potential variability of the portfolio allocation, which is likely an important consideration for most retirees.

While the unique facts and circumstances associated with each client should dictate the appropriate distribution portfolio, it is likely that an overly aggressive portfolio (say 100/0) will prove to be too risky for most clients, while an overly conservative portfolio with a higher Success to Variability ratio (say 0/100) will not provide an adequate probability of success when addressing the likelihood of longer distribution periods. Therefore, based on the research conducted for this paper, as well as other qualitative and practical considerations, the optimal allocation for most retirees is likely a balanced portfolio, such as a 60 percent equity and 40 percent fixed income/cash allocation.

JFP

Endnotes

1. Social Security Administration Web site: http://www.ssa.gov/OACT/STATS/table4c6.html.
2. Life Expectancy by Age, 1850–2004, Web site: http://www.infoplease.com/ipa/A0005140.html.
3. Data definitions:

   Intermediate-term bond. Defined as the return on the Moody's Seasoned Aaa Corporate Bond Yield, assuming a ten-year duration. Data obtained from the St. Louis Federal Reserve: http://research.stlouisfed.org/fred2/.

   Cash. Defined as the yield on the 3-Month Treasury bill. Secondary Market Rate, data obtained from Tradetools.com (1927–1933) and the St. Louis Federal Reserve (1934–2006): http://research.stlouisfed.org/fred2/.

   Domestic large-blend equity. Defined as the return on the "Big Neutral" portfolio based on the 2 × 3 portfolio return information publicly available on Kenneth French's Web site: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

   International equities. Defined as the return on the Global Financial Data World ex-USA Return Index, data obtained from Global Financial Data.

   Since pure historical data is used for this analysis, as is common among distribution research, the author would caution the reader that if future returns are lower than historical returns, the actual result of a distribution portfolio may be materially different from what this research suggests.
4. Data obtained from the Bureau of Labor Statistics: http://data.bls.gov/PDQ/servlet/SurveyOutputServlet.
5. The equity reduction for the concave allocations was determined using the following formula: ((Previous Year's Equity Allocation) – ((Distribution Year)^2)*.00002)). All allocations were rounded to the nearest whole percentage.
6. The convex allocation formula was determined using the mirror image of the concave hyperbola.
7. Credit for this portion must be given to Dr. Gregory W. Kasten, who after working with the author on the paper, recognized the importance of combining both the probability of success (or failure) and the underlying variability of the portfolio and suggested such a metric.
8. Familiarity was one of the original reasons (along with cost and convenience) that Markowitz (1959) selected variance (or standard deviation) as the definition of risk (as opposed to semi-standard deviation, which also referred to as downside risk). Familiarity and convenience are two of the primary reasons for its continued popularity, despite its noted shortcomings. See, for example, Nawrocki (1999), Sortino and Satchell (2001), and Swisher and Kasten (2005).
9. For readers not familiar with the Sharpe ratio, it is a reward-to-variability (or risk) calculation that is a common method for comparing portfolios and investments on a risk-adjusted basis. The calculation is the excess return (return minus the risk-free rate) divided by the standard deviation.
10. Based on the demographic information obtained from the CFP Board Web site (updated as of 01/31/2007): http://www.cfp.net/media/profile.asp.
11. Assuming that the investment fees increase the necessary real withdrawal rate as opposed to decreasing the real returns of the portfolio.

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