Appendix A: Algorithms for the Recognition and Detection of Trend Changes

Recognition

The recognition approach uses values of the S&P 500 Index adjusted for inflation starting with January 1903.[1]

1. For month n, tabulate the S&P 500 Index value (SP_n) and the Consumer Price Index value (CPI_n) for the beginning of the month.
2. For months prior to January 1903, skip to step 6. For month n starting with January 1903, calculate the trailing 12-month average of the CPI.
   - $CPI_{n\_12\text{-}mo\_avg} = (\mathbf{SUM}\ (i = n - 11 \text{ to } i = n)\ CPI_i) / 12$
3. For month n starting with January 1903, calculate the annual percent change in the $CPI_{n\_12\text{-}mo\_avg}$ from the time one year prior.
   - $CPI_{n\_ann\_pct\_cx} = 100 \times ((CPI_{n\_12\text{-}mo\_avg} / CPI_{n-12\_12\text{-}mo\_avg}) - 1)$
4. Calculate the average (geometric mean) overall monthly inflation factor based on the percent change in the $CPI_{n\_12\text{-}mo\_avg}$ over the last year ("^" signifies exponent, as in Excel formulas).
   - $I_{n\_mo\_fctr} = (1 + (CPI_{n\_ann\_pct\_cx} / 100)) \text{ ^ } (1 / 12)$
5. Calculate the one-month inflation adjustment factor, used to calculate the inflation-adjusted value of an amount in month n in terms of the previous month's dollars.
   - $I_{n\_mo\_adj} = 1 / I_{n\_mo\_fctr}$
6. The overall inflation adjustment factor ($I_{n\_overall\_adj}$) is used to calculate the inflation-adjusted value of an amount in month n in terms of the value in the month prior to the start of the series of adjustments, in this case, in terms of December 1902 dollars. For months prior to January 1903, set $I_{n\_overall\_adj}$ equal to 1. For month n starting with January 1903, calculate $I_{n\_overall\_adj}$ to use to convert the S&P 500 Index value to December 1902 dollars.
   - $I_{n\_overall\_adj} = I_{n\_mo\_adj} \times I_{n-1\_overall\_adj}$
7. For month n, calculate the adjusted S&P 500 Index value.
   - $SP_{n\_adj} = I_{n\_overall\_adj} \times SP_n$
8. For month n, calculate the trailing 12-month average of the adjusted S&P 500 Index values.
   - $SP_{n\_12\text{-}mo\_avg} = (\mathbf{SUM}\ (i = n - 11 \text{ to } i = n)\ SP_{i\_adj}) / 12$
9. For month n, calculate the monthly percent change in the 12-month average.
   - $SP_{n\_adj\_cx} = 100 \times (SP_{n\_12\text{-}mo\_avg} - SP_{n-1\_12\text{-}mo\_avg}) / SP_{n-1\_12\text{-}mo\_avg}$
10. Calculate the sums of the $SP_{n\_adj\_cx}$ for month n plus from one to five previous months (SP_{sum_-1}, SP_{sum_-2}, ..., SP_{sum_-5}).
    - $SP_{sum\_-1} = SP_{n\_adj\_cx} + SP_{n-1\_adj\_cx}$
    - $SP_{sum\_-2} = SP_{n\_adj\_cx} + SP_{n-1\_adj\_cx} + SP_{n-2\_adj\_cx}$
    - ... to $SP_{sum\_-5} = SP_{n\_adj\_cx} + SP_{n-1\_adj\_cx} + SP_{n-2\_adj\_cx} + SP_{n-3\_adj\_cx} + SP_{n-4\_adj\_cx} + SP_{n-5\_adj\_cx}$
11. If the $SP_{n\_adj\_cx}$ or any of the sums from step 10 is 3 percent or greater for a shift from bearish to bullish or is −3 percent or less (is more negative) for a shift from bullish to bearish, a transition has occurred starting with the month prior to the earliest month for which a change was included in the adjusted change or in the sum.

Detection

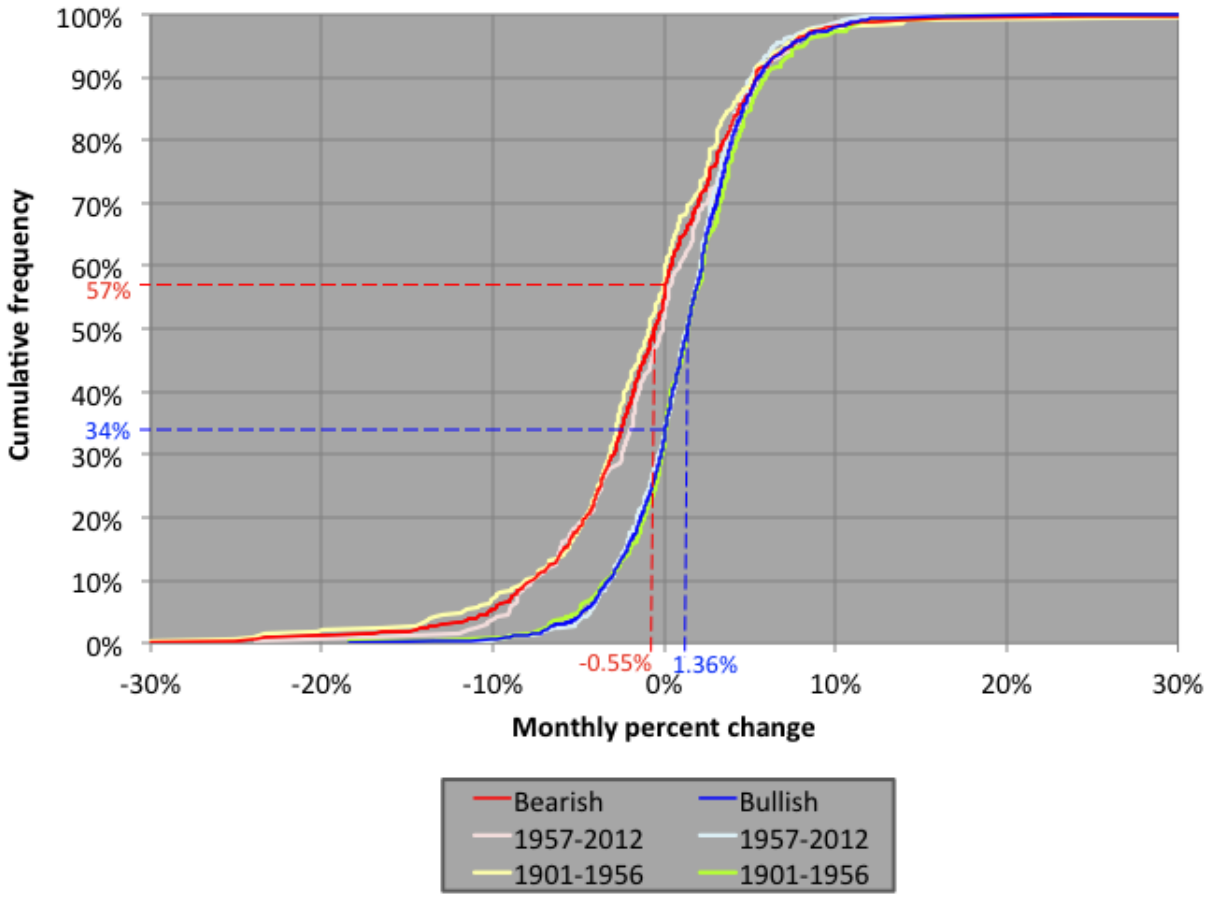
1. Construct cumulative frequency curves of the historical monthly changes in the S&P 500 Index and in the CPI, as illustrated in Appendix B for the S&P 500 Index, separately for bullish and bearish periods as defined by the recognition procedure. The method of generating statistical cumulative distributions as well as where to obtain historical data for the CPI and the S&P 500 Index (SPX) are presented in the previous publication (Cohen 2011a).
2. Use random-generated numbers to project the values of the S&P 500 Index and of the CPI forward month-by-month up to five months into the future from the present month, as described in endnote 9 of the paper. This entails five projections forward of the SP 500 and seven of the CPI, since the current SP 500 Index value is available at the beginning of a month, but the CPI becomes available only two months after the month to which it applies.
3. For one "run," apply the recognition algorithm described above for each forward-projected month. If the recognition algorithm results in recognition of a transition at some projected month, that month becomes a "projected recognition" month.
4. For 5,000 runs, tabulate how many of them result in a recognition in each of the forward-projected months (=N), and calculate the percentage of such positive runs as $100 \times N / 5000$.
5. If the percentage for any of the forward-projected months reaches the stipulated criterion threshold (for example, 60 percent, 75 percent, or 90 percent, as described in the paper), the criterion for a detection at that threshold is reached.
6. For a 60 percent or 75 percent threshold, confirm a detection by simultaneous or subsequent reaching of a 90 percent threshold within four months of the detection at the lower percent criterion.
7. In all cases, further confirm detections (and lack of transitions) by ongoing checking for recognitions.

Appendix B: Statistical Distributions Used for Projections and Transition Detection

Monthly S&P 500 Index Changes

The historically derived frequency distributions for bullish and bearish periods of the month-to-month percent changes of the S&P 500 Index are displayed in Figure B1. The fact that the bearish (red) curve is to the left of the bullish (blue) curve reflects the expected tendency of these statistical trends. The farther the curve is to the left, the more generally bearish the market statistical behavior described by the curve is. The historical median (50^{th} percentile) for bearish trends is −0.55 percent change per month (range: −29.9 percent to +37.7 percent); 57 percent of the bearish-trend monthly changes were zero or negative, whereas only 43 percent were positive. Similarly, the bullish curve being farther to the right reflects the more bullish behavior. The bullish historical median monthly percent change is 1.36 percent (range: −18.4 percent to +40.7 percent); 66 percent of the monthly changes during a bullish period were positive, whereas only 34 percent were zero or negative.

Figure B1: S&P 500 Index Monthly Percent Changes

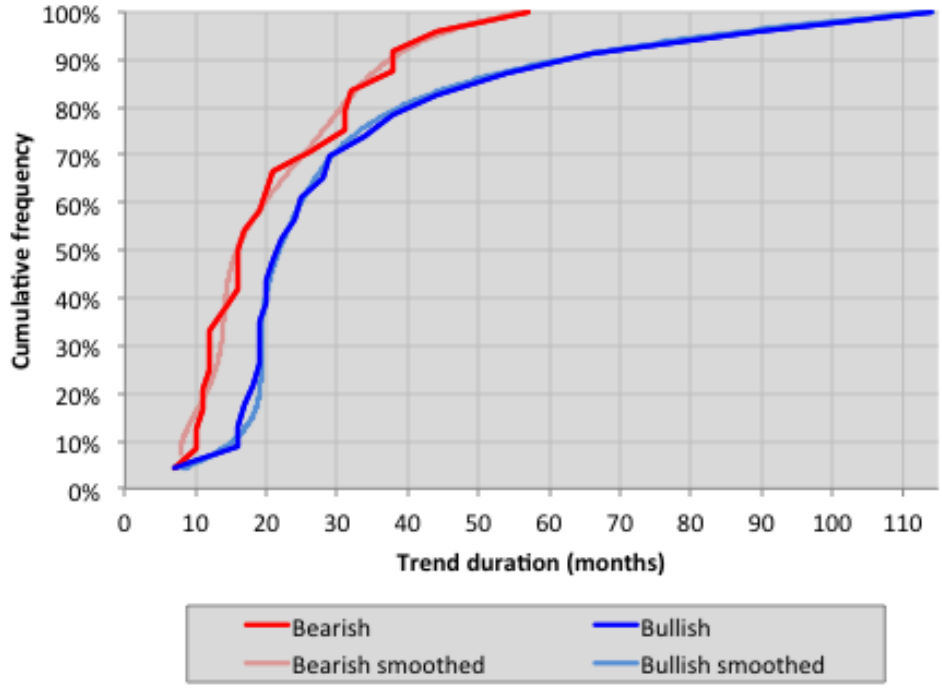


To test for temporal stability of these distributions, changes in the S&P 500 Index distributions incorporating only the first half and the last half of the century were compared to each other and to the full-century distribution. As is evident in Figure B1, the light-colored half-century plots are in close proximity and partially overlap with each other and with the corresponding overall red and blue plots (in many places the light plots are not seen, due to being overlain by the overall plots or by each other). Thus, the differences in the cumulative-frequency profiles are small, and they are not statistically significant.[2] This indicates that the distributions are highly stable despite using data as disparate in time as possible. When one considers the variegated history of the world over these time periods, the level of stability in these cumulative-frequency profiles is remarkable.[3] Nevertheless the caveat must be stated that the stability of these distributions into the future cannot be considered entirely certain.

Trend Durations

The cumulative-frequency distributions of trend durations are displayed in Figure B2. (The individual durations also are tabulated for each trend period in Table 2 in the paper.) As indicated by the bearish distribution being to the left of the bullish distribution, the bearish trend durations tend to be shorter, ranging from seven to 57 months (median: 16); the bullish trend durations range from seven to 114 months (median: 21.5).

Figure B2: Historical Trend Durations

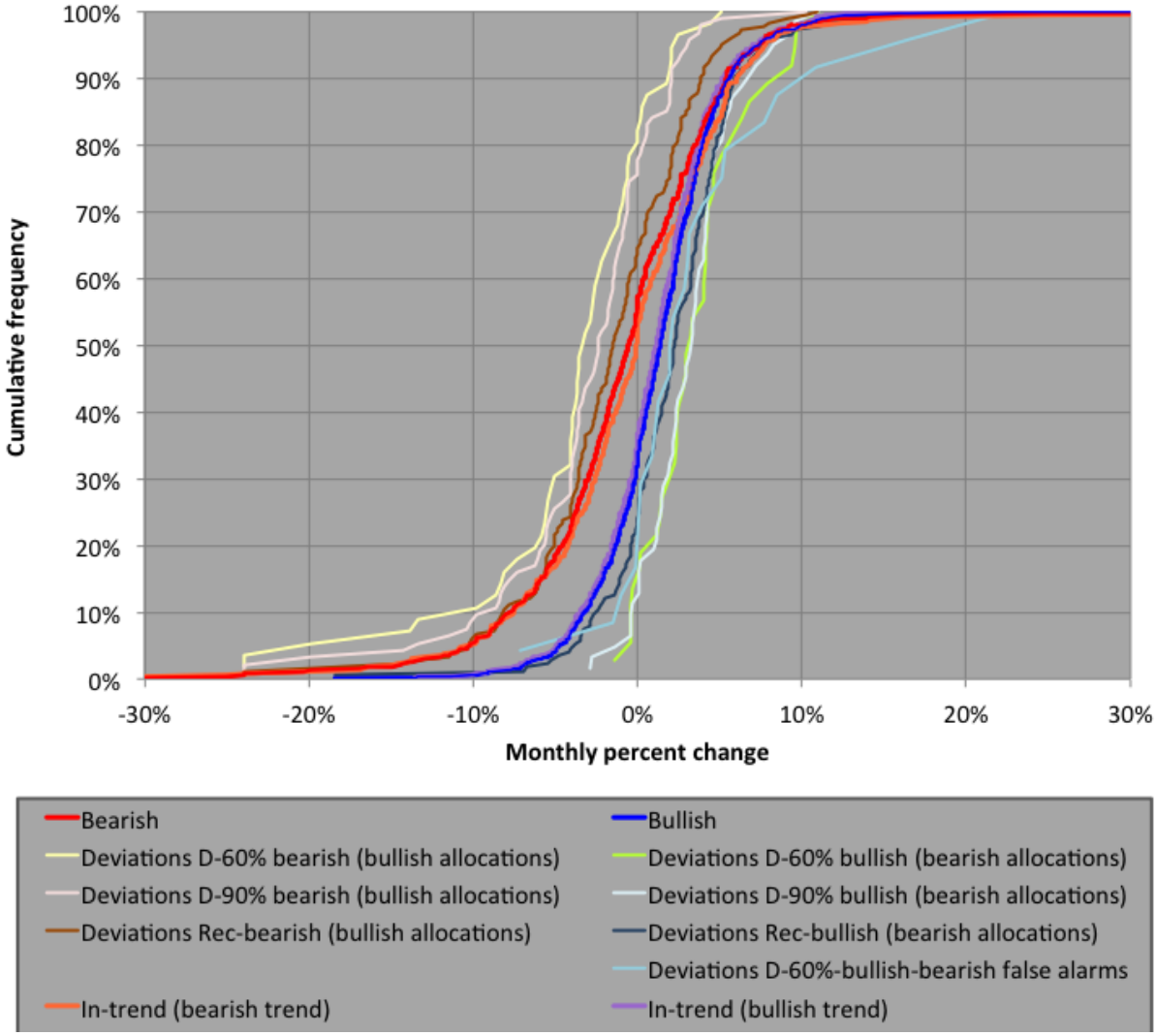


Note: Cumulative frequency distributions of the CPI value percent changes were used to project the percent changes in the running 12-month average of the adjusted S&P 500 Index; these distributions are not shown.

Appendix C: Behavior of Trends – Exaggerated at Beginnings and Trend-Biased Throughout

"Deviation periods" are those in which allocations would be those appropriate for the opposite trend from the existing trend (that is, bullish during a bearish trend or vice versa). Most deviation periods are when a delay occurs in a detection or recognition from the time of the actual transition, but some are rare instances when an allocation switch is made (1) before transitions after detections prior to transitions (see Table 2 in the paper) and (2) after false alarms. Figure C1 shows that the deviation periods tend to have exaggerated characteristics of their respective actual trend periods. Thus, the times at the beginnings of bullish trends during which bearish allocations would still be in place before detection or recognition of the transition are more bullish than the general bullish trends, and the times at the beginnings of bearish trends during which bullish allocations would still be in place before detection or recognition of the transition are more bearish than the general bearish trends.[4]

Figure C1: S&P 500 Index Monthly Percent Changes – Deviations and In-Trend Changes



The bullish and bearish plots in Figure C1 representing the general trends are the same as those in Figure B1 in Appendix B. The exaggerations, reflected in the deviation-sample bearish plots being further to the left than the general bearish plot and in the deviation-sample bullish plots being further to the right than the general bullish plot, are less overall for the recognitions. The recognition plots include periods further into the overall trend period – further from the recognized transition. This exaggeration of trends near transition months makes especially important the detection of the trend changes as early after the actual transition as possible, so allocation strategies can be changed as early as possible to take advantage of the new, initially exaggerated trends.

The monthly changes represented in Figure C1 in the "in-trend" distributions are those within trends, but excluding the deviation periods for the recognition approach (that is, starting from six to eight months after the beginnings of the trends, based on when each trend was recognized). The in-trend bullish plot is nearly coincident with (and thus somewhat hidden by) the general bullish plot. That these plots are similar to those for the full trends, and statistically significantly different from each other for the bearish and bullish trends (see next paragraph and Table C1) demonstrates that the biased statistical behavior of the market during the trends continues well within the trend periods. The biased behavior

during trends is not dependent solely on the exaggerated behavior at the beginnings of trends (represented by the deviation plots).

Table C1 shows results of statistical tests showing significance of the differences in distributions of monthly changes during the deviation periods versus during the corresponding bullish or bearish periods exclusive of these months and between in-trend bearish versus bullish periods.[5] The same tendency toward exaggeration of the bullish trend appears to be the case for the difference for false alarms, although it did not reach statistical significance (possibly associated with the smaller sample of false-alarm deviation months). False alarms can be seen as reflecting a tendency toward new-trend-beginning periods without quite making it to a full transition.

Table C1: Kolmogorov-Smirnov (K-S) Statistical Test Results

K-S test results	Bullish			Bearish		
	N	D	p	N	D	p
Recognition deviations	657 & 174 → 831	17.3%	<0.001**	340 & 181 → 521	14.9%	0.009**
D-90% deviations	769 & 62 → 831	30.8%	<0.001**	427 & 94 → 521	29.3%	<0.001**
D-60% deviations	794 & 37 → 831	32.4%	0.001**	465 & 56 → 521	31.5%	<0.001**
D-60% false alarms deviations	807 & 24 → 831	19.4%	0.313			
	Bearish vs Bullish					
In-trend changes	340 & 657 → 997	19.5%	<0.001**			
Notes: *p ≤ 0.05, **p ≤ 0.01, for two-sided significance tests.						

The exaggerations of market behavior near beginnings of trends are due to the requirement for trend transitions to be identified by monthly market changes meeting a minimum criterion in the direction of the new trend. In contrast, trend continuation is associated with *absence* of a transition to a trend in the other direction, not continuation of meeting of the minimum criterion. Therefore, fluctuations during a trend that are opposite to the trend's direction, but that do not meet the criterion for a trend in the opposite direction, are included in the statistics for the given trend. This results in less-strong behavior in the direction of the trend overall than during the criteria-driven beginnings of the trends. Nevertheless, this somewhat weaker trending behavior during the trends than at the beginnings is associated with and essential to the intermediate-duration nature of the trends, allowing trend-appropriate allocations to be maintained for long enough to be advantageous overall. The continued reality and significance of the trend-biased market behavior during the identified trends exclusive of their beginnings is demonstrated by the in-trend plots in Figure C1 and the in-trend comparison included in Table C1.

Appendix D: Results of the Statistical Comparisons of the Frequency Distributions of Estate Projections

The frequency distributions of estate projections are presented in Figure 3 in the paper, for different criteria to indicate trend transitions. The results of statistical comparisons of these distributions are presented in Table D1. The results of the D-60 percent strategy were distributed significantly to higher values than those of the D-90 percent strategy. Neither D-90 percent nor D-60 percent had results distributed statistically differently from those of the D-75 percent strategy. Although the difference in distribution of results was not significant for the recognition method versus the no-slow-timing method, the results at the higher end of the recognition distribution clearly diverge higher and approach those for the detection methods.

In Table D1, probability values for a one-sided test were used, based on the expectation of results progressively better in the order of no-slow-timing, recognition, D-90 percent, D-75 percent, and D-60 percent, based on the numbers of deviation months in the historical analyses; the one-sided *p* values are estimated as half those for the two-sided test (see www.uni-muenster.de/ZIV.BennoSueselbeck/s-html/helpfiles/ks.gof.html; K-S tests were administered using the online tool at www.physics.csbsju.edu/stats/KS-test.html). N for all comparisons was 300 for each sample.

Table D1: Results of Kolmogorov-Smirnov Tests Comparing Distributions for Results of Estate Projections

Comparisons	Recognition	D-90%	D-75%	D-60%
No slow-timing	D = 8.3% p = 0.119	D = 13.3% p = 0.005**	D = 16.7% p < 0.001**	D = 16.7% p < 0.001**
Recognition		D = 11.0% p = 0.025*	D = 13.7% p = 0.004**	D = 19.3% p < 0.001**
D-90%			D = 5.0% p = 0.419	D = 11.3% p = 0.020*
D-75%				D = 8.7% p = 0.100
Notes: *p ≤ 0.05, **p ≤ 0.01, for one-sided significance tests.				

Appendix Endnotes

[1] For January 1901 through December 1902, the unadjusted values for the SP 500 Index were used; this is equivalent to using a value of 1 for the overall inflation adjustment factor ($I_{n\_overall\_adj}$, calculated in step 6 for subsequent months). Note that to get current overall inflation adjustment factors for converting to December 1902 dollars, the calculations need to be made starting for January 1903, using CPI values back to February 1901 to get the trailing 12-month averages of the CPI starting in January 1902.

[2] A Kolmogorov-Smirnov significance test was performed to compare the 1901-to-1956 and the 1957-to-2012 distributions; the small differences were not found to be statistically significant (bearish: D = 8.8%, p = 0.25, N = 284 and 237; bullish: D = 6.1%, p = 0.42, N = 389 and 435). The same test indicates a statistically highly significant difference between the overall bearish and bullish curves (D = 24.3%; $p < 0.001$, N = 521 and 824). The tests were done using an online tool at www.physics.csbsju.edu/stats/KS-test.html.

[3] These results were unexpected and, in our opinion, warrant further research examining different historical time slices of differing slice durations.

[4] The samples showing exaggerated behavior overall include rare instances of early switching at the ends of bullish trends after early detection of transition to bearish trends and early switching during bearish trends after early detection of transition to bullish trends.

[5] The statistical tests in Table C1 were done using an online tool at www.physics.csbsju.edu/stats/KS-test.html.