Appendix 1: Bond Yield Model

The simulation begins with an initial interest rate (the seed value), which is set to 2.5 percent, which is the approximate yield to maturity of the Barclays Aggregate Bond Index as of January 2014. The yield for each subsequent year (Y_t) is based on equation 1.1, which is an autoregressive model with a single lag (the previous year's yield), where ε_Y is independent white noise with a mean of 0 percent and a standard deviation that varies. The yield is assumed to be a minimum of 1.0 percent and a maximum of 10.0 percent.

$$Y_t = \alpha_Y + \beta_Y Y_{t-1} + \varepsilon_Y \qquad [1.1]$$

The values for α_Y, β_Y, and the standard deviation for ε_Y vary based on the nominal return assumptions, and are noted in Table A1.

Table A1: Coefficients for Equation 1.1 for Different Nominal Return Assumptions

	α_Y	β_Y	σ for ε_Y
Low	.250%	.300%	.300%
Mid	.925	.950	.960
High	1.000%	1.000%	2.000%

After the annual bond yields have been determined, the total return for cash is determined using equation 1.2, where α_c = -3.024 percent, β_c = .978, Y_t = the yield for that year, $\beta_{y\Delta c}$ = .321, $Y = Y_t - Y_{t-1}$, and ε_c is an independent white noise with a mean of 0 percent and a standard deviation of 1.0 percent. The total return for cash is assumed to be a minimum of 0 percent and a maximum of 10 percent.

$$r_c = \alpha_c + \beta_{yc} Y_t + \beta_{y\Delta c} \Delta Y + \varepsilon_c \qquad [1.2]$$

The next step is to determine the return for bonds, stocks, and inflation, based on equation 1.3. The coefficient values for equation 1.3 for bonds, stocks, and inflation are included in Table A2.

$$r_i = \alpha_i + \beta_{yi} Y_t + \beta_c r_{c,t} + \beta_{y\Delta i} \Delta Y + \varepsilon_i \qquad [1.3]$$

Table A2: Coefficients for Bonds, Stocks, and Inflation

	α_i	β_{yi}	β_c	$\beta_{y\Delta i}$	σ for ε_i	Min	Max
Bonds	.900%	.678	.446	-3.714	5.066%	-15%	40%

Stocks	7.951%	-.308	.593	-4.221	19.358%	-100%	200%
Inflation	2.983%	.964	-.554	1.012	2.088%	-10%	20%

The α_i value for stocks is decreased by 2.0 percent for the low nominal return scenario and increased by 2.0 percent for the high nominal return scenario. The α_i value for inflation is decreased by .5 percent for the low inflation scenario and increased by 1.0 percent for the high inflation scenario.

Appendix 2: Preference Model Methodology

This section introduces the model used to estimate shortfall risk and residual wealth.

Shortfall Risk

The first step to estimate shortfall risk is determine the percentage of the target income goal replaced during each year of each simulation run, as noted by equation 2.1. Hypothetical retirees at various initial starting ages (t =1) and model retirement periods up to 50 years (T = 50) are considered. While using the percentage of the income replaced may seem like an unnecessary step, versus using the actual amount of income received, determining the percentage of the goal (liability replaced) allows the shortfall risk measure to be reviewed in the cost of the residual wealth (the bequest).

$$IRP_t = \frac{SS_t + AI_t + PI_t}{N_t} \qquad [2.1]$$

For equation 2.1, IPR_t is the percentage of the target income replaced in simulation run year (t), SS_t is the annual pension/social security income, AI_t is the total annual annuity income, PI_t is the annual income from the portfolio, and N_t is annual income goal (need).

Next the weighted average income replacement for a given run is determined, as noted in equation 2.2. In this step, opposed to focusing on the income replacement in a given year (year 20 of retirement), the income replacement during each year is included, based on a discount rate and the probability of either of the retirees being alive at that year. This allows enables each year to be included in the calculation versus focusing on a single year. There is always an assumed social security benefit to ensure the income replacement value is not equal zero.

$$WIRP_r = \sum_{t=1}^{T} \frac{cp_t^s(1+\beta_r)^{-t}\,\frac{(IRP_t)^{1-\gamma}}{1-\gamma}}{\sum_{t=1}^{T} cp_t^s(1+\beta)^{-t}} \qquad [2.2]$$

In equation 2.2, $WIRP_r$ is the weighted average income replacement percentage for a given run (r), p_t^s is the cumulative probability of surviving to year (t), β_r is the real discount rate (a real discount rate is used because the income goal increases by inflation) and is assumed to be 2 percent, IPR_t is the percentage of the target income replaced in simulation run year (from equation 2.1), and γ is risk aversion coefficient. The moderate risk preference scenario assumes a risk aversion coefficient of 4, while the low risk preference scenario assumes a risk aversion coefficient of 2, and the high risk preference scenario assumes a risk aversion coefficient of 8. shortfall risk is the certainty equivalent value for each of the $WIRP_r$ runs, as noted by equation 2.3.

$$Shortfall\ Risk = 1 - \left(\left(\left(\frac{\sum_{r=1}^{R} WIRP_r}{R} \right) (1 - \gamma) \right)^{\left(\frac{1}{1-\gamma}\right)} \right) \qquad [2.3]$$

Residual Wealth

residual wealth is the average discounted and mortality weighted assets that would pass to the hypothetical retiree's heirs, divided by the total cost of retirement (i.e., the liability). This formula is noted in equation 2.4, where cp_t^d is the probability of dying by year t, β_n is a nominal discount rate based on the expected return of the non-Annuity portfolio before fees,Port_t is the value of the portfolio at year t, Ann_t is the mortality weighted net present value of future guaranteed annuity benefit payments (if applicable) at year t, discounted at 2.0 percent for inflation-adjusted payments and 4.0 percent for nominal payments.

$$Residual\ Wealth = \frac{\sum_1^R \dfrac{\dfrac{cp_t^d (1+\beta_n)^{-t} (\mathrm{Port}_t + \mathrm{Ann}_t)}{\sum_{t=1}^{T} cp_t^d (1+\beta_n)^t}}{\dfrac{cp_t^s (1+\beta_r)^{-t} N_{r,t}}{\sum_{t=1}^{T} cp_t^s (1+\beta_r)^t}}}{R} \qquad [2.4]$$

Total Benefit

Once shortfall risk and residual wealth have been estimated, it is possible to determine the total benefit received from each scenario. This is done using equation 2.5, where p^* varies between 0 and 1 based on the retiree's preference for bequest. A p^* value of 0 indicates a low bequest preference (a retiree only focused on shortfall risk); a value of .5 indicates a moderate preference, and a value of 1 would indicate a low preference.

$$Benefit = (1 - Shortfall\ Risk) + (p^* * \ Residual\ Wealth\ ) \qquad [2.5]$$

Appendix 3: Mortality Modeling

Mortality is modeled using the "Gompertz Law of Mortality;" which is named for Benjamin Gompertz. Gompertz discovered that a person's probability of dying increases at a relatively constant exponential rate as age increases. The formulation of Gompertz's law for mortality is that used by Milevsky (2012), where the probability of survival to age *t*, conditional on a life at age (*a*), is given by equation 3.1.

$$q_t = \exp \left\{ \exp\left\{\frac{a-m}{b}\right\} \left(1 - \exp\left\{\frac{t-a}{b}\right\}\right) \right\} \qquad [3.1]$$

where, m is the modal lifespan and b is the dispersion coefficient. The Gompertz parameters are "fitted" to the Society of Actuaries 2000 Annuity Table by minimizing the sum of squared differences from the parameters and the actual mortality estimates in the actual Table. The model lifespan for males is estimated to be age 88 with a dispersion coefficient of approximately 10.65 years; a modal lifespan for females is age 91 with a dispersion coefficient of approximately 8.88 years. Independence in life expectancies is assumed for a joint couple (male and female) and probability of at least one member surviving to age *t* is estimated using equation 3.2.

$$q_t^{Joint} = 1 - ((1 - q_t^{Male})(1 - q_t^{Female})) \qquad [3.2]$$