A Broader Framework for Determining an Efficient Frontier for Retirement Income

by Wade D. Pfau, Ph.D., CFA

William Bengen’s seminal 1994 article on sustainable withdrawal rates in the Journal of Financial Planning provided a much needed reality check on popular retirement discourse by demonstrating how the sequence of returns risk causes the sustainable withdrawal rate from a portfolio of volatile assets to fall below the average return to those assets. Bengen described the SAFEMAX, which he defined as the sustainable withdrawal rate from the worst-case scenario in history. It was closer to 4 percent than to numbers like 7 percent bandied about in the media at that time. Bengen’s research answered an important question about sustainable spending rates. Several years later, Cooley, Hubbard, and Walz (1998) published a study popularly known as the Trinity study. It introduced a small but significant modification to Bengen’s work. Rather than reporting the historical worst-case scenario, the Trinity authors calculated success rates and corresponding failure rates for different withdrawal rates and asset allocation strategies over differing retirement durations. Based on the U.S. historical data since 1926, success rates are the percentage of rolling historical periods in which some financial wealth remained at the end. Financial wealth depletion becomes synonymous with a failed retirement in this framework, as seen, for instance, when Terry (2003) wrote, “I believe
that most investors would find even a 1 percent probability of failure to be excessively high when dealing with irreplaceable assets and considering the extreme costs of failure.” Perhaps it was an unintended consequence, but the research on safe withdrawal rates that has followed from these notions provides a rather incomplete picture of retirement income.

The idea that retirees should focus on finding a spending strategy that maintains a rather low failure rate using a diversified portfolio over a fixed retirement period, as is standard with safe withdrawal rate studies, is not adequate. Problems abound, including:

1. Failure rates do not consider the retiree’s entire balance sheet of assets for income generation. With Social Security and other defined-benefit pensions, some clients may find that financial asset depletion is not disastrous.

2. Failure rates are not compatible with financial products designed to provide lifetime retirement income, such as single-premium immediate annuities (SPIAs). Such annuities are assets and the present value of their remaining lifetime income payments can be included on a personal balance sheet, but this value is not a part of financial assets. Failure rates are not meaningful for clients seeking to understand the implications of partial annuitization.

3. Failure rates ignore the lost potential enjoyment from spending more early in retirement. They are an extreme outcome measure that puts weight only on financial wealth depletion. Client spending potential is irrelevant. Clients must find an appropriate personal balance between the aims of spending more and then having to make potentially larger subsequent cutbacks in the event of a long life and a sequence of poor market returns. According to Milevsky and Huang (2011), the decision about this tradeoff depends both on longevity risk aversion (the fear of outliving financial assets) and the amount of “pensionized” income from outside the financial portfolio. Working toward a similar end, Finke, Pfau, and Williams (2012) show how clients may potentially tolerate a higher failure rate to maximize their spending power and overall lifetime enjoyment in retirement.

4. Failure rates ignore the magnitude and severity of the “failure” condition. For how long and by how much will a client’s income fall short of what is desired or needed? Failure is failure regardless of whether spending falls short by $1 or by $100,000.

5. The failure rate framework is based too heavily on the U.S. historical record since 1926. It doesn't take into proper consideration that circumstances may change and current conditions may suggest a different starting point for today’s retirees. Failure rates seemingly justify the use of historical averages for Monte Carlo simulations rather than current market conditions. Why else would it be relevant to a client today if a 5 percent withdrawal rate worked for someone retiring in 1935, for instance? Planners and clients need to think more broadly beyond failure rates when developing their retirement-income strategies. The objective in this paper is to outline characteristics of a broader retirement-income framework and to provide an example of its use with an efficient frontier for retirement income.

Literature Review of Four Generations of Retirement Studies

Tresidder (2012) provides an overview of the history of safe withdrawal rate research, in which he classifies three generations of past studies. The first generation simply assumes an average annual portfolio return, providing withdrawal rate guidance based on that. The second generation, developed by Bill Bengen, incorporates the sequence-of-returns risk associated with volatile portfolios and finds that historically a more conservative 4 percent withdrawal rate is much closer to being safe. Third-generation researchers generally seek to incorporate more realism into second-generation results by including factors such as market conditions at the retirement date, fees, longevity beyond 30 years, and an acknowledgment that the post-1926 U.S. historical data is not sufficient to provide much confidence about the viability of the 4 percent rule. The third generation clarifies how 4 percent is only an educated guess based on limited historical data and some rather simplifying assumptions. They argue that there are other ways to view and interpret the historical data, and retirees only have one opportunity to get things right in retirement.

A growing body of research now exists that could be considered the fourth generation of studies. This research broadens the safe withdrawal rate question to place it into the wider context of a complete retirement income strategy that can include forms of annuitization. Ameriks, Veres, and Warshawsky (2001) was one of the first studies to investigate the beneficial impact of partial annuitization with a fixed single premium immediate annuity on portfolio sustainability. Chen and Milevsky (2003) also examine the optimal allocation between immediate annuities and other financial assets. Milevsky (2009) further develops his product-allocation framework. He builds a retirement-income frontier in which users can aim to maximize spending power for an acceptable level of retirement sustainability and expected...
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other outcome measures. For instance, different income tools using a variety of cation by simulating the performance of the median legacy value.

reduce income risk while also raising that partial annuitization can potentially single-period returns. They emphasize lifelong retirement income rather than portfolio theory, but for the case of ing efficient frontier as found in modern that allow them to create a correspond-

provide the risk and return measures of remaining wealth at age 86, which is 25 percent chance to survive to against “Financial “Probability of financial wealth depletion at age 92, which the 65 year old has a 25 percent chance to survive to) against “legacy potential” (the median amount of remaining wealth at age 86, which is the life expectancy for the 65-year-old male). Income risk and legacy potential provide the risk and return measures that allow them to create a correspond-
ing efficient frontier as found in modern portfolio theory, but for the case of lifelong retirement income rather than single-period returns. They emphasize that partial annuitization can potentially reduce income risk while also raising the median legacy value.

Several other studies from 2012 also investigate questions about product allo-
cation by simulating the performance of different income tools using a variety of other outcome measures. For instance, Tomlinson (2012) uses a loss-aversion utility function to compare spending shortfalls and bequest values for an inflation-adjusted SPIA, systematic withdrawals from portfolios of stocks and bonds, and partial annuitization approaches. He finds that depending on how a client views the tradeoff between bequests and spending shortfalls, optimal income strategies side toward either all stocks or all SPIAs.

Malhotra (2012) also seeks to develop guidelines for a comprehensive frame-
work to evaluate retirement-income strategies. He defines two reward and three risk metrics, which are summary measures to analyze different income strategies. Reward measures are average income and average legacy, while risk measures are the probability of failure, the magnitude of failure, and the percentage of income from fixed sources. He incorporates several case studies to show how clients can evaluate the tradeoffs and seek solutions. His framework already incorporates the Social Security claiming decision (the age at which to begin claiming Social Security benefits), bond ladders, and time segmentation.

Finally, Pfau (2012) builds a framework to analyze eight retirement income strategies, including constant and variable withdrawal rate strategies, single-premium immediate annuities, and variable annuities with guaranteed lifetime withdrawal benefit riders. Outcome measures emphasize downside risk, upside potential, and bequests, and the entire distribution of outcomes is shown. As each strategy is simulated in isolation, issues of product allocation are not directly explored.

Methodology

Retirement Income Planning Process. Retirement income planning is a process that should be agnostic in the choice of products and withdrawal approaches. The performance of various combinations of income tools can be simulated and optimized to individual client circumstances.

Product Allocation. Though not comprehensive, retirement-income tools considered here will follow the product-allocation framework defined by Milevsky. Clients may rely on systematic withdrawals from a portfolio of stocks and/or bonds invested with a total-returns perspective, or they may allocate some of their financial assets at retirement to buy inflation-adjusted SPIAs, SPIAs with fixed nominal payments, or a VA with a GLWB rider.

Consistent Fees. It is important to maintain a consistent fee structure among the various retirement income tools to avoid biasing results, as the compounding effects of fees over time can be dramatic and create large biases about optimal strategies.

The analysis here is based on realistic low-cost versions for the various income tools available in the marketplace. For systematic withdrawals, I assume that investors use low-cost index funds, and the stock and bond funds are assumed to have a 0.2 percent annual fee. SPIA prices are from Vernon (2012), who obtained them using the Income Solutions platform at the start of April 2012. The 65-year-old couple can buy a 100 percent joint-and-survivors SPIA with a payout rate of 3.875 percent for the inflation-adjusted version and 5.84 percent for the fixed version. Pricing aspects of the VA/GLWB include a payout rate of 4.5 percent, annual fees on the VA contract value of 0.6 percent, an annual guarantee rider fee of 0.95 percent on the high-watermark benefit base, and an annual step-up feature to increase payouts if the contract value reaches a new high watermark. The assumed asset allocation for the VA/GLWB is 70 percent stocks and 30 percent bonds. Using low-cost versions for each tool allows for more direct and meaningful comparisons. Further advisory fees could
returns than in the past, as the corre-
assumptions call for lower U.S. stock
rather than historical averages. The
connected to current market conditions
since 1926, the return assumptions are
calibrated to the U.S. historical data
standard deviations and correlations
each product allocation strategy. While
200 Monte Carlo simulations used for
asset markets that guide the underlying
Table 1 provides the assumptions for
assets. But the current reality is that
interest rates increase in five to 10 years,
or today’s retirees to derive benefit than it is for today’s savers. As
well, to provide a comparable basis for
annuity prices, which are based on cur-
rent bond yields, it is important to have
matching assumptions for systematic
withdrawals. This explains why current
bond yields serve as a basis for the
underlying assumptions in this research.

Retirement Financial Goals. As
argued, retirees should not be narrowly
focused on a singular goal to avoid
financial wealth depletion. Financial
goals for retirement can essentially be
reduced to two competing objectives:
1. Support minimum spending needs
and lifestyle-spending goals as best as
possible, however high they may be.
2. Maintain a reserve of financial
assets to support risk-management
objectives such as protection
from expensive health shocks,
divorce, unexpected needs of other
family members, severe economic
downsarts, etc., or to meet legacy
objectives.

To the extent that greater spending
requires more financial resources,
and that more aggressive investment
strategies imply greater upside wealth
as well as greater downside spending
risks, these objectives generally must be
balanced in a manner that best supports
client preferences.

Utility measures translate spending
into values that incorporate the concept
of diminishing marginal enjoyment
from spending increases. But defining
an appropriate utility function for
clients may be a difficult task. As an
alternative here to operationalize how
well a strategy meets spending goals,
I propose summing the amounts of
spending shortfalls created by a strategy.
With Monte Carlo simulations for
financial market returns, each strategy

Table 1: Assumptions for Real Asset Returns

<table>
<thead>
<tr>
<th></th>
<th>Arithmetic Mean</th>
<th>Geometric Mean</th>
<th>Standard Deviation</th>
<th>Stocks</th>
<th>Bonds</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks</td>
<td>5.1%</td>
<td>3.1%</td>
<td>20.0%</td>
<td>1.0</td>
<td>0.1</td>
<td>-0.2</td>
</tr>
<tr>
<td>Bonds</td>
<td>0.3%</td>
<td>0.1%</td>
<td>7.0%</td>
<td>0.1</td>
<td>1.0</td>
<td>-0.6</td>
</tr>
<tr>
<td>Inflation</td>
<td>2.1%</td>
<td>2.0%</td>
<td>4.2%</td>
<td>-0.2</td>
<td>-0.6</td>
<td>1.0</td>
</tr>
<tr>
<td>Equity Premium</td>
<td>4.8%</td>
<td></td>
<td></td>
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</tbody>
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Note: Standard deviations and correlation coefficients are based on Stocks, Bonds, Bills, and Inflation data provided by Morningstar and Ibbotson Associates, in which the U.S. S&P 500 index represents the stock market and intermediate-term U.S. government bonds represent the bond market. The arithmetic mean for bond returns is calibrated to recent TIPS yields. The arithmetic mean for inflation is based on the breakeven inflation rate implied by TIPS and Treasury yields. The arithmetic mean for stock returns is calibrated to allow an equity premium of 4.8% above the bond return, which is the equity premium for a GDP-weighted portfolio of 19 developed market countries between 1900 and 2010 from the Dimson, Marsh, and Staunton Global Returns Dataset provided by Morningstar and Ibbotson Associates.
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regardless of any hypothetical minimum be viewed by the client as a failure goal, as spending less than this may focus on spending below the lifestyle enclosure, though, may be more inclined to Planners with practical client experi-

A minimum level to meet basic needs. What could be truly disastrous is if their spending is forced to fall below a needs and lifestyle goals. Clients may their lifestyle goals unsustainable. But what could be truly disastrous is if their spending is forced to fall below a minimum level to meet basic needs.

Though scalable, each client requires a personalized efficient frontier matching household characteristics including age, gender, marital status, health, life expectancy, desired spending patterns, and constraints about asset and product allocation (such as a restriction that no more than 50 percent of assets will be annuitized).

Customized expectations about the returns, volatilities, and correlations for asset classes and inflation, as well as mortality, can also be included, as can details about the current pricing of available retirement income tools. Different clients will desire different withdrawal rates for meeting basic expenses and for lifestyle-spending goals, and they will also have varying amounts of financial assets and varying inflation-adjusted income and fixed income from sources outside of the financial portfolio. After an efficient frontier of product allocations is constructed for these characteristics, clients can select one of the points on the frontier reflecting their personal preferences about the tradeoff between meeting spending goals and maintaining sufficient financial reserves. Results will be presented for a 65-year-old heterosexual couple (this distinction is relevant as gender affects survival probabilities) who are retiring and have already claimed Social Security (incorporating optimal claiming decisions into the framework will be left for subsequent research).

At retirement, assets are divided between stocks, bonds, inflation-adjusted SPIAs, fixed SPIAs, and VA/ GLWBs. Each of these five components is simulated in 10 percentage point increments from zero to 100 percent, with 1,001 possible product allocations. For example, a product allocation of 20, 30, 10, 30, 10 means that at the retirement date, 20 percent of assets are invested in stocks, 30 percent in bonds, 10 percent into a real SPIA, 30 percent into a fixed SPIA, and 10 percent into a VA/GLWB. In this case, half of the assets remain in the financial portfolio with a fixed-asset allocation of 40 percent stocks and 60 percent bonds, as these represent the relative shares of the total assets devoted to stocks and bonds. For the portfolio of financial assets, annual rebalancing restores the fixed asset allocation.

To obtain each year’s spending amount (which is withdrawn at the start of each year), the maximum allowable amount is taken first from the annuitized financial products. If this sum exceeds what is needed to meet the lifestyle spending goal, then the excess funds are added to the financial port-
folio. If the guaranteed income sources do not provide enough income to meet the lifestyle goal, then the remainder needed is withdrawn from the financial portfolio (stocks and bonds). Whenever fewer assets remain in the financial portfolio (stocks and bonds) than are needed to meet the spending goal, the client is unable to meet his or her full lifestyle-spending goal for that year. If that occurs, the client can only spend the income available from any annuities and Social Security. The contract value of the VA/GLWB rider is included as part of the financial assets plotted on the y-axis, but clients do not withdraw more than the allowable amount each year so as to avoid breaking the contract terms for the guarantee.

A single-premium immediate annuity allocation of 100 percent can still support a reserve of financial assets if the income received from the SPIA exceeds the lifestyle spending goal, as the excess is reinvested. In the case where the initial allocations to stocks and bonds are both zero, income returned to the investment portfolio in this manner is allocated to 100 percent stocks. Lifestyle goals and minimum needs are defined in real terms, while fixed SPIAs and VA/GLWBs only provide nominal guarantees (GLWB step-ups cannot generally be expected to keep pace with inflation). As inflation erodes the real value of the nominal income, the percentage of lifestyle goals these sources support declines.

Survival Probabilities. The outcome measures—remaining financial assets and percentage of lifetime spending needs which are satisfied—are calculated using survival probabilities from the Social Security Administration 2007 Period Life Table. The remaining financial-assets measure is the sum of remaining financial assets in each year of retirement, times the probability that the second member of the couple dies in that year. With this measure, the real value of remaining financial assets at death cannot be zero, as some retirees will die shortly after retirement when assets will surely remain. This measure is an ex ante estimate of remaining financial assets based on survival probabilities for an unknown lifetime. Meanwhile, the percentage of lifetime-spending needs that are satisfied is calculated as the sum of spending in each year as a percentage of the minimum spending need, times the probability that at least one member of the couple survives to that age.

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Distribution of Outcomes. In presenting outcomes, the part of the distribution of outcomes that should be highlighted is not clear. For financial assets, should we focus on the mean amount, the median amount, or the remaining amount in a bad luck outcome such as the 10th percentile for values ranked from lowest to highest across the simulations? Likewise, the Monte Carlo simulations will produce a distribution of percentages of minimum spending needs that are met. Should we focus on the mean or median percentage of met needs across the simulations, or instead identify the percentage of needs met in a bad luck case such as the 10th percentile? Different possibilities should be investigated, though the issue becomes less important if the optimal strategies are fairly consistent between these different ways of measuring remaining financial assets and percentages of spending needs that are satisfied.

Results

Results are provided for a 65-year-old couple. Their Social Security benefit is equal to 2 percent of their retirement assets. Their lifestyle-spending goal is 6 percent of retirement assets, which requires them to use a 4 percent withdrawal rate above Social Security to meet their goal. Minimum needs are also 6 percent. Social Security is relevant because increasing the Social Security benefit will help support a higher percentage of spending needs being met, as it provides a further guaranteed income floor even when financial assets are depleted. Though higher Social Security benefits will not change the product allocations found on the efficient frontier, it will shift the frontier to the right, which could increase client comfort with choosing more aggressive allocations with a greater probability to increase financial asset holdings.

The efficient frontier for this case is shown in Figure 2, with the median value of remaining financial assets at death shown on the y-axis, and the 10th percentile bad-luck outcome of spending needs that could be satisfied shown on the x-axis. The figure illustrates how 1,001 different product allocations perform in meeting the two objectives. Moving toward the upper right hand corner of the figure is advantageous for retirees. A purple curve is added to show all of the product allocations consisting only of financial assets (stock and bonds). These outcomes represent some of the worst possible, demonstrating a clear role for partial annuitization to improve retiree outcomes, even in today’s low interest-rate environment. In
particular, the red curve highlights most of the points on the efficient frontier. It shows the allocations between stocks and fixed single-premium immediate annuities. The results will vary when changing the personal characteristics and spending goals of the client. For instance, if spending needs require a withdrawal rate of only 1 percent of assets, then it is likely that spending needs will be met with any product allocation and the client can focus on the allocation that maximizes the median value of remaining financial assets: 100 percent stocks. But in the particular case of Figure 2, the evidence suggests that optimal product allocations consist of stocks and fixed SPIAs, and clients need not bother with bonds, inflation-adjusted SPIAs, or VA/GLWBs. Though SPIAs do not offer liquidity, they provide mortality credits and generate bond-like income without any maturity date, and they support a higher stock allocation for remaining financial assets. Altogether, this allows a client to better meet both retirement financial objectives. Any of the product allocations on the efficient frontier represents a potentially optimal point, and clients can choose which one they think best balances their own objectives. Of course, any product allocations the client is not comfortable using could be removed and a constrained efficient frontier could be determined instead. For instance, clients may fear high stock allocations—especially if they may be tempted to sell after a market drop—or too much annuitization.

An interesting result is that fixed SPIAs dominate inflation-adjusted SPIAs in the retirement portfolio for this example. In personal correspondence, Joseph Tomlinson explains why this may be. Essentially, either because of a lack of competition or because of the difficulties of hedging inflation risks, inflation-adjusted SPIAs are not priced competitively with fixed SPIAs. Fixed SPIAs supply more inflation-adjusted income than real SPIAs until cumulative inflation sufficiently reduces the real value of the fixed SPIA income stream. With payout rates of 5.84 percent, the fixed SPIA payout is 51 percent larger than the 3.875 percent payout of the inflation-adjusted version. With inflation fluctuating around 2.1 percent, it takes almost 20 years, on average, for the income from the inflation-adjusted SPIA to grow larger. Because using retirement-date assets to buy a fixed SPIA provides more income than needed until inflation sufficiently reduces the real value of the SPIA payments, and because excesses are invested in a portfolio of 100 percent stocks, a 100 percent allocation to a fixed SPIA can support more of the spending needs and result in the same median amount of financial reserves.
as a portfolio of 40 percent stocks and 60 percent bonds. Nevertheless, it is important to emphasize that clients with particular concerns about inflation may expect that the breakeven inflation rate is too low and may seek the additional protection provided by an inflation-adjusted SPIA.

**Conclusion**

Further refinements to this framework are needed to include a broader range of retirement-income strategies. These include optimization for the Social Security claiming decision, bond ladders and time segmentation approaches, delayed or laddered annuity purchases, deferred income annuities (longevity insurance), immediate variable annuities, long-term care insurance, life insurance, reverse mortgages, home equity loans, structured products based on derivatives to protect on the downside while maintaining some upside, and various other products. Equally important, taxes must be added to the analysis so that the efficient frontier can be calculated in after-tax terms. More detailed scenarios with varying asset-class assumptions, personalized client circumstances, and changing patterns for spending needs also can help to determine which sorts of findings are generalizable across a wide variety of circumstances. The approach described here provides the initial stages of a framework that can better inform planners and clients, and guide them in the direction of optimal retirement income strategies.

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**References**


